Bicycle wheel flop: labs in development

The torque and the runaway: theory, instructions, and data sheets

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The drop lab measures the flop factor ζ_{eff} of a bicycle as a length: how far the front end sinks as the bars turn. The same quantity can be measured as a torque, with a spring scale, and as a growth rate, by releasing the bars and filming them. These two measurements are described here as *labs in development*: the theory is settled and the procedures are written, but neither has been run with students, each has an apparatus question that the drop lab does not, and the expected numbers are predictions from the geometry, not measurements. This document is independent of the drop lab, but it assumes the geometry of the bicycle has been measured as described there (α, R, ρ, o, and the trail check), and it refers to the drop lab's value of ζ_{eff} as the one the two methods are checked against. Part I gives the theory, with the assumptions of the model stated as plainly as I can and the additional assumptions the torque and the runaway bring with them. Part II gives the instructions and the open questions. Derivations are in the paper the labs are drawn from.[1]

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[1] N. Tufillaro, "Bicycle wheel flop," submitted to Am. J. Phys. (2026). Equation numbers in brackets refer to that paper.

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Part I
Theory

1 The geometry

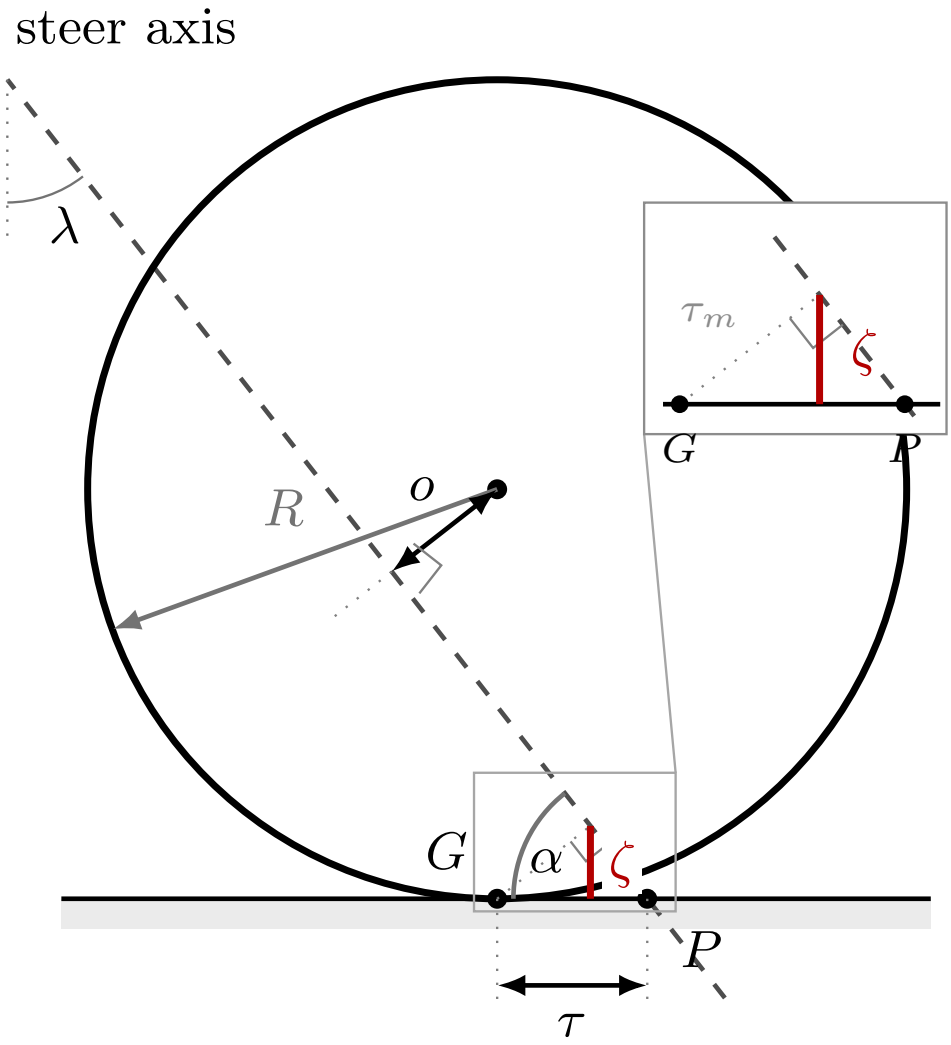


Figure 1: The symbols of Table 1, drawn with an exaggerated head angle so that the offset and the trail are legible (Fig. 1 of the paper). The steer axis meets the floor at P; the tire touches at G. The ground trail τ is GP along the floor, the mechanical trail τ_m is the perpendicular from G to the axis, and the flop factor ζ (red, enlarged in the inset) is the height at which that perpendicular meets the axis.

Table 1: Geometric symbols. "Measured" means read directly from the bicycle (see the drop lab, Sec. 3); "computed" means obtained from the measured ones by the equation given.

Symbol	Meaning	How obtained	Typical (road)
α	head angle, steer axis to the floor	measured, angle gauge on the head tube	73°
$\lambda = 90^\circ - \alpha$	steer-axis tilt from vertical	computed	17°
R	loaded radius, floor to front axle	measured, rule	0.340 m
ρ	half the inflated tire width	measured, caliper	14 mm
o	fork offset, axle to steer axis, perpendicular	measured, two rules	45 mm
w	wheelbase, axle to axle	measured, tape	1.0 m
τ	ground trail, G to P along the floor	computed, Eq. (1); checked on the floor	57 mm
τ_m	mechanical trail, G to the axis, perpendicular	computed, Eq. (2)	54 mm
ζ	flop factor, knife-edge wheel	computed, Eq. (3)	15.9 mm
ζ_{eff}	flop factor with the tire	computed, Eq. (4)	14.7 mm
δ	steer angle from straight ahead	measured, floor protractor	0–75°
δ_*	flop angle, where released bars rest	computed, Eq. (6); observed	63°

$$\tau = \frac{R\cos\alpha - o}{\sin\alpha} \qquad \text{[paper Eq. (2)]} \tag{1}$$

$$\tau_m = \tau\sin\alpha = R\sin\lambda - o \qquad \text{[Eq. (3)]} \tag{2}$$

$$\zeta = \tau_m \sin\lambda = \sin\lambda\,(R\sin\lambda - o) = \tau\sin\alpha\cos\alpha \qquad \text{[Eq. (9)]} \tag{3}$$

$$\zeta_{\text{eff}} = \sin\lambda\,\big[(R-\rho)\sin\lambda - o\big] \qquad \text{[Eq. (10)]} \tag{4}$$

Equation (4) is Eq. (3) with R replaced by $R-\rho$: a fat tire flops like a thin wheel whose radius reaches only to the middle of the tread. All three experiments measure ζ_{eff}, and the geometric value from Eq. (4) is the prediction they are compared with.

The drop itself, the quantity the drop lab measures, is

$$\Delta h(\delta) = (R-\rho)\,(1-\cos\gamma) - o\sin\lambda\,(1-\cos\delta), \qquad \sin\gamma = \sin\lambda\,\sin\delta, \qquad \text{[Eqs. (7), (8), (10)]} \tag{5}$$

with γ the camber angle of the leaned wheel, and for small angles $\Delta h \simeq \zeta_{\text{eff}}(1-\cos\delta) \simeq \frac{1}{2}\zeta_{\text{eff}}\delta^2$. The flop angle, where the drop is largest, is

$$\cos\delta_* = \frac{\beta\cos\lambda}{\sqrt{1-\beta^2\sin^2\lambda}}, \qquad \beta = \frac{o}{(R-\rho)\sin\lambda}. \qquad \text{[Eq. (13)]} \tag{6}$$

Both labs below are ways of measuring $d(\Delta h)/d\delta$ rather than Δh itself.

2 The model and its assumptions

The drop law, Eq. (5) below, is a statement about geometry, not about forces, and it pays to be explicit about what is being assumed, because the assumptions are exactly the things the laboratory setup has to enforce.

Two rigid bodies and a hinge. The bicycle is treated as two rigid bodies: the *frame* (with the rear wheel, which plays no part), and the *front assembly* (fork, handlebars, and front wheel), hinged to the frame along the steer axis. The steer angle δ is the one coordinate of that hinge. The wheel is a rigid torus: a tube of radius ρ whose centerline is a circle of radius $R-\rho$ about the axle. Nothing bends, nothing squashes.

The frame is held. A rigid body has six degrees of freedom. The model fixes five of them for the frame: it may not roll (lean sideways), yaw (turn about a vertical axis), or pitch (nod about a horizontal axis), and it may not move horizontally. It may move *vertically*, and that is the only freedom it has. In the drop lab a bottom-bracket stand removes roll, yaw, and horizontal motion, and leaves the frame free to pitch by a fraction of a degree, which is accounted for there by a lever factor. In the two labs below the bicycle stands on its wheels and is held upright by a helper at the saddle, or sits in the same stand; how well the frame is held, and whether the helper applies steering torque without meaning to, is one of the open questions.

The steer axis is fixed in space. Because the frame's attitude is fixed, the steer axis is a fixed line, tilted back from vertical by λ in the bicycle's plane of symmetry. Turning the bars by δ therefore rotates the whole front assembly rigidly about that fixed line. Everything about the front wheel follows from this one rotation: the hub moves around the axis on a circle of radius o, dropping by $o \sin\lambda(1-\cos\delta)$ relative to the axis, and the wheel plane, which contained the vertical at $\delta = 0$, leans by the camber angle γ with $\sin\gamma = \sin\lambda\sin\delta$. The axle is horizontal only at $\delta = 0$; at $\delta = 30°$ it is tilted by 8.4°. Nothing is "adjusted" to keep the frame upright; the frame is held upright, and the wheel's lean is the consequence.

The floor supplies one constraint. The floor is a rigid horizontal plane, and the only condition it imposes is that the lowest point of the tire lie on it: $z_{\min} = 0$. Here is the counting that makes the drop a kinematic certainty rather than something to be solved for. Once δ is chosen, the front wheel's position relative to the frame is completely determined (the hinge has one coordinate and it has been set), so the height of the wheel's lowest point relative to the frame is a definite function of δ. The frame has exactly one freedom left, its height. The floor's one condition uses up that one freedom. There is nothing left to choose: the frame *must* sit at the height that puts the tire's lowest point on the floor, and $\Delta h(\delta)$ is simply the amount that height differs from its value at $\delta = 0$. The frame drops because, with the wheel leaned and the hub swung, the tire's lowest point is closer to the frame than it was, and the floor pushes the frame down to meet it.

The lowest point of a leaned torus is easy to find. Let $\mathbf{c}$ be the hub and $\mathbf{n}$ the unit normal to the wheel plane (the axle direction) after the rotation. The direction in the wheel plane that points most steeply down is $\mathbf{w} = (\hat{\mathbf{z}} - (\mathbf{n}\cdot\hat{\mathbf{z}})\mathbf{n})/|\hat{\mathbf{z}} - (\mathbf{n}\cdot\hat{\mathbf{z}})\mathbf{n}|$, and the contact point is

$$\mathbf{G}(\delta) = \mathbf{c}(\delta) - (R-\rho)\,\mathbf{w}(\delta) - \rho\,\hat{\mathbf{z}}. \tag{7}$$

The height of the lowest point below the hub is $(R-\rho)\cos\gamma + \rho$, which is where the first term of Eq. (5) comes from.

The contact point is not stationary. Nothing above says the tire touches the floor at a fixed spot; it touches wherever its lowest point happens to be, and Eq. (7) says that spot moves. For the example geometry it walks about 0.9 mm per degree of steer, mostly *across* the wheel (toward the inside of the lean): 18 mm at 20°, 50 mm at 60°. This has no effect on the drop, which depends only on the lowest point's height, but it matters for how the tire is supported. A wheel cannot roll sideways, so on a fixed floor a stationary bicycle's tire must *scrub* across the surface as the bars turn. The drop lab avoids the scrub by lifting the tire and placing it fresh at each angle. The two labs below cannot: the tire must carry its load while the bars move, so the contact point has to be free to travel, which is what the floating plate of Sec. 8 provides.

Rolling is irrelevant. The wheel is a surface of revolution about its axle. Spinning it about the axle moves no point of the surface anywhere new, so whether the wheel rolls, slides, or is locked makes no

difference to where its lowest point is. The model does not assume rolling, and does not assume the wheel is prevented from rolling; it does not need to say anything about it.

Friction is not in the drop, but it is in the torque. Because $\Delta h(\delta)$ is fixed by geometry alone, friction at the contact cannot change it: friction can resist the motion of the bars, but it cannot put the frame anywhere other than the one height the constraints allow. (The exception is a flexible tire, which friction can distort by a fraction of a millimeter; see Sec. 6.) Friction does enter the *torque*. The flop torque in the paper is the conservative torque $T = -dV/d\delta$ with $V = N_f\, h$, which is the whole steering torque only if the floor does no work as the contact point moves, i.e. if the tire moves without sliding friction. That is true for a rolling bicycle, where the sideways drift of the contact point is accommodated by the rolling path curving, and it is why the torque measurement (Sec. 8) must support the tire on something that lets the contact point move freely.

Quasi-static. The bars are turned slowly, so the drop is a sequence of equilibrium positions and no inertia enters. The runaway measurement (Sec. 9) is the dynamic version, where the same $V(\delta)$ drives the bars away from center at a rate set by the steering inertia.

What is left out, and what it costs. Tire squash and fork flex (both make the measured drop a few percent smaller than Eq. (5), never larger); the frame's pitch on the stand (up to 9% at δ_* on a bottom-bracket stand, half that on two wheels, nothing at small angles); headset friction (resists the turning, no effect on the height); the weight of the front assembly (no effect on the drop, which is pure geometry; it adds a small torque in the torque lab). Table 2 collects the assumptions and says what in the laboratory enforces each one.

Table 2: The model's assumptions as they apply to the drop, how a laboratory enforces them, and what happens if they fail. The additional assumptions of the torque and runaway are listed in Sec. 3.

Assumption	Enforced by	If violated
Rigid frame and fork; rigid torus tire	stiff bicycle, tire at rated pressure	drop reads low by a few percent (squash, flex)
Frame does not roll	stand cradle; bubble level across the top tube	odd-in-δ term; cancels in the L/R average
Frame does not yaw or move horizontally	stand fixed on tape marks; protractor centered on P	δ misread; P and the pointer disagree
Frame does not pitch; head angle fixed	*not* enforced by a bottom-bracket stand: frame pitches by $\Delta h/x_G$ (or by $\Delta h/w$ on two wheels)	dial reads x_{cap}/x_G of the drop (corrected by the lever factor); head angle steepens by $\leq 0.5^\circ$ (up to 9% at δ_*)
Frame attitude = on-road attitude	angle gauge on the head tube reads α in the stand	ζ_{eff} off by 16% per degree
Floor is a rigid horizontal plane	hard smooth platform, shimmed level	tire sinks into a soft surface; drop reads low
Tire's lowest point is on the floor; contact point free to be anywhere	lift-turn-set-down, so the tire is placed, not scrubbed	scrub winds up the sidewall; sub-mm offsets, hysteresis between L and R approaches
Quasi-static	bars turned slowly, read at rest	frame bounces; dial unsettled

3 Additional assumptions for the torque and the runaway

The drop is pure kinematics. The torque and the runaway are dynamics, and they need more.

The floor does no work. The flop torque is defined as the conservative torque $T = -dV/d\delta$ with $V = N_f\, h(\delta)$, the potential energy of the front-wheel load as the frame rises and falls. That is the whole torque about the steer axis only if the floor's force on the tire does no work as the contact point travels. On a rolling bicycle this is true: the sideways drift of the contact point is taken up by the rolling path curving, and there is no sliding. On a stationary bicycle it is false unless the tire is supported on something that moves with the contact point, which is what the floating plate of Sec. 8 is for. Any residual friction adds a torque that opposes the motion and is therefore *odd* under reversing the direction of approach; approaching each angle from both sides and averaging removes it, and the difference measures it.

The load is constant. N_f is taken as fixed while the bars turn. It is measured once, with the bars straight, on a bathroom scale under the front tire. The frame's small pitch as the front end drops moves a little weight forward, under 1%; a helper leaning on the saddle moves a great deal more, and so does a rider shifting in the seat.

The fork has weight. The front assembly's center of mass hangs below and ahead of the tilted steer axis, and gravity on it adds a torque of the same sign as the flop, with stiffness $g\, m_A u_A \sin\lambda$ near center [Eq. (17)]. The model of the drop ignores this because it does not affect the height; the torque and runaway cannot. It is about 8% of $N_f \zeta_{\text{eff}}$ for the paper's benchmark bicycle with a rider and a much larger fraction for a bare bicycle, which is why the loaded run gives the cleaner ζ_{eff} and why the bare-bicycle runaway rate quoted below includes it explicitly.

Quasi-static (torque) and small-angle, undamped (runaway). The torque is read with the bars at rest, so no inertia enters. The runaway is the opposite limit: the bars are released and the only torques are the gravitational ones, with the equation of motion linearized about $\delta = 0$, $I_\delta \ddot{\delta} = k_{\text{tot}}\delta$. That requires δ small enough that T is still proportional to δ, in practice below about 25°, and it neglects headset friction, the rolling resistance of the plate, and air drag on the wheel, all of which slow the growth. It also takes I_δ, the moment of inertia of the front assembly about the steer axis, as a constant, which it is as long as nothing on the fork moves.

The frame is held still. In the runaway, the frame must not move: if it is free to lean or yaw, the rear of the bicycle has to be accelerated too and the growth is slower than Eq. (10) predicts. A helper holding the saddle, or the bottom-bracket stand, provides this; a helper who applies steering torque through the saddle, however slight, changes the release.

4 The torque

The floor's push on the front tire produces a torque about the steer axis

$$T(\delta) = N_f \sin\lambda\, \sin\delta \left[\frac{(R-\rho)\sin\lambda\cos\delta}{\cos\gamma} - o \right], \qquad k \equiv \left.\frac{dT}{d\delta}\right|_{\delta=0} = N_f\, \zeta_{\text{eff}}, \qquad \text{[Eq. (12)]} \tag{8}$$

where N_f is the load on the front wheel. The torque is positive (pushes the bars away from center) for $0 < \delta < \delta_*$, zero at δ_*, and negative beyond it. The spring scale measures the torque needed to hold the bars, which is $-T$; the data sheet records F and the direction of pull, and $T = \pm FL$ with the sign fixed by the direction.

The spring scale also feels the weight of the front assembly itself, which hangs below the tilted steer axis and adds a torque of the same sign. Near center the total gravitational stiffness is

$$k_{\text{tot}} = N_f\, \zeta_{\text{eff}} + g\, m_A u_A \sin\lambda, \qquad \text{[Eq. (17)]} \tag{9}$$

where m_A is the mass of the front assembly (fork, wheel, bars, stem) and u_A the perpendicular distance of its center of mass from the steer axis. For the paper's benchmark bicycle the second term is 8% of the first; for a bare bicycle with no rider (N_f small) it is a much larger fraction, which is why the loaded run, with a rider aboard, gives the cleaner value of ζ_{eff}. The term $m_A u_A$ can be estimated by hanging the removed front assembly from its steerer (Fig. 8) and measuring how far its center of mass settles from the axis, or simply by taking the difference between the measured k_{tot} and $N_f \zeta_{\text{eff}}$ from the drop lab.

Table 3: Torque: what is measured and what is extracted.

Symbol	Meaning	In the experiment
N_f	load on the front wheel, N	bathroom scale under the front tire, $\times g$
L	lever arm, steer axis to the hook	rule, perpendicular to the axis
F	spring-scale force, perpendicular to the bar	average of the two approaches at each δ
$T = FL$	steering torque	computed
k_{tot}	gravitational stiffness, N m/rad	slope of T vs δ (radians) through the origin
ζ_{eff}	flop factor	$(k_{\text{tot}} - g\, m_A u_A \sin\lambda)/N_f$, or k_{tot}/N_f as an upper bound
δ_*	flop angle	the δ at which F changes sign

Analysis. Convert δ to radians. Plot T against δ with the sign convention above, and fit a line through the origin to the 10° and 20° points (and 30° if the curve is still nearly straight); the slope is k_{tot}. Then plot Eq. (8) using the measured N_f and geometry, plus the constant-slope term $g\, m_A u_A \sin\lambda \cdot \delta$ if $m_A u_A$ is known, and compare the whole curve, including the sign change at δ_*. The difference between the two readings at each angle, divided by two, is the rolling friction of the floating plate (or thrust bearing) plus the headset friction; record its size.

5 The runaway

For small steer angles the front assembly, released with the frame held upright, obeys

$$I_\delta\, \ddot{\delta} = k_{\text{tot}}\, \delta, \qquad \delta(t) = \delta_0 \cosh \sigma t, \qquad \sigma = \sqrt{\frac{k_{\text{tot}}}{I_\delta}} = \sqrt{\frac{N_f \zeta_{\text{eff}} + g\, m_A u_A \sin\lambda}{I_\delta}}, \tag{10}$$

where I_δ is the moment of inertia of the front assembly about the steer axis, and δ_0 is the release angle. After the first few tenths of a second $\cosh \sigma t \simeq \frac{1}{2} e^{\sigma t}$, so a plot of $\ln \delta$ against t becomes a straight line of slope σ. The small-angle law holds only while δ is below about 25°; beyond that the torque of Eq. (8) falls off and the bars decelerate toward δ_*.

The steering inertia comes from the torsion pendulum,

$$I_\delta = I_p\, \frac{T_1^2}{T_2^2 - T_1^2}, \qquad I_p = \tfrac{1}{2}\, m_p\, (r_1^2 + r_2^2), \tag{11}$$

where T_1 is the period of the assembly alone, T_2 the period with a plate of mass m_p and inner and outer radii r_1, r_2 added on the axis, and I_p the plate's inertia about its own axis. The wire's torsional stiffness cancels out.

Table 4: Runaway: what is measured and what is extracted.

Symbol	Meaning	In the experiment
$\delta(t)$	steer angle versus time	read from the video every 4 frames (1/60 s at 240 fps)
δ_0	release angle	the first frame after the finger lets go
σ	growth rate, s^{-1}	slope of $\ln\delta$ vs t on the straight part
T_1, T_2	torsion periods without and with the plate	ten oscillations each, timed
m_p, r_1, r_2	plate mass and radii	balance, caliper
I_δ	steering inertia	Eq. (11)
$k_{\mathrm{tot}} = \sigma^2 I_\delta$	gravitational stiffness	computed; compare with the torque measurement
ζ_{eff}	flop factor	$(k_{\mathrm{tot}} - g\,m_A u_A \sin\lambda)/N_f$

Analysis. For each release, tabulate t and δ, take $\ln\delta$, and fit a straight line to the points between the first clear motion and $\delta = 25°$. Average σ over the releases; the scatter gives its uncertainty. Then $k_{\mathrm{tot}} = \sigma^2 I_\delta$, which should agree with the slope from the torque measurement. The complete bicycle, free to lean and yaw, runs away more slowly than the front assembly alone because more mass has to be accelerated; holding the frame upright by the saddle, as Sec. 9 directs, removes that coupling and makes Eq. (10) the right description.

6 Expected values

For the road geometry of Table 1 with $N_f = 45\,\mathrm{N}$ (bare bicycle) the stiffness is $k = N_f\zeta_{\mathrm{eff}} = 0.66\,\mathrm{N\,m/rad}$, the torque at 10° is $0.12\,\mathrm{N\,m}$, and the maximum, near 33°, is $0.25\,\mathrm{N\,m}$: a pull of $1.3\,\mathrm{N}$ on a 20 cm arm. With a rider, $N_f = 310\,\mathrm{N}$, multiply by 6.9: $k = 4.6\,\mathrm{N\,m/rad}$ and a maximum of $1.8\,\mathrm{N\,m}$. If $I_\delta = 0.10\,\mathrm{kg\,m^2}$ and $g\,m_A u_A \sin\lambda = 0.3\,\mathrm{N\,m/rad}$, both estimates for a typical road bicycle rather than measurements, the runaway rate is $\sigma = \sqrt{(0.66 + 0.3)/0.10} = 3.1\,\mathrm{s}^{-1}$ bare and $\sqrt{(4.56 + 0.3)/0.10} = 7.0\,\mathrm{s}^{-1}$ with a rider: released from 3°, the bare bicycle's bars pass 25° in about 0.9 s. The main uncertainties: N_f must be measured under the conditions of the run; the friction torque must be shown to be small compared with $k\delta$ at 10° ($0.12\,\mathrm{N\,m}$ bare), which is the hardest requirement in the torque lab; and the torsion periods must be timed to better than 1%, because I_δ depends on the difference $T_2^2 - T_1^2$.

Part II
Experiment

7 Measuring the steer angle δ

Both experiments need the steer angle, the angle through which the front assembly has been turned from straight ahead. The simplest scale is a paper protractor on the floor (Fig. 2). Print a 360° protractor about 25 cm across, cut it out, and tape it to the floor centered on P (the point where the steer axis meets the floor, found by laying a rule along the head tube down to the floor) with its 0° line along the bicycle's centerline. Tape a pointer, a straightened paper clip or a strip of stiff card, to the back of the fork crown so that it hangs just above the paper and points along the wheel. The bicycle's centerline is the line through the two tire contact marks; use the long rule to draw it on the tape before placing the protractor.

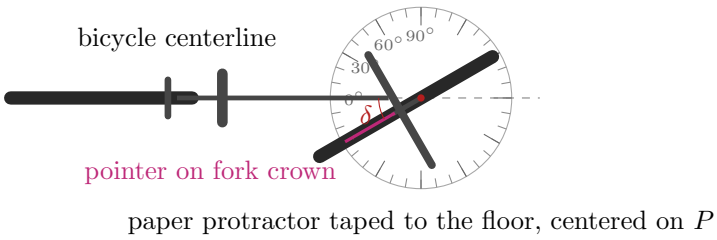


Figure 2: Top view. A paper protractor is taped to the floor, centered on P with 0° along the centerline, and a pointer fixed to the fork crown reads the steer angle δ. Turning the bars to the left (as drawn) is taken as positive.

Two cautions. First, as the bars turn the front end drops, so the pointer must clear the paper at $\delta = 0$ by a few millimeters. Second, the angle you want is the angle of the front wheel, not of the handlebars; they are the same thing on a rigid bicycle, but a loose headset or a bent stem makes them differ. Reading at the fork crown avoids the question. A reading to ±1° is good enough.

8 Torque measurement

Idea

Gravity pushes the turned bars away from center with a torque $T(\delta)$ whose slope at the origin is $k = N_f \zeta_{\text{eff}}$, where N_f is the weight on the front wheel. The torque is measured with a spring scale pulling on the end of the handlebar; T is the force times the distance from the steer axis to the point of pull.

Two things that make this measurement hard

The tire does not want to pivot in place. The friction of a loaded contact patch is of order 1 N m, larger than the flop torque near center, so on a bare floor the bars simply stick. Nor is a pivot all that is needed. As the bars turn, the point where the tire touches the floor moves: by about 0.9 mm per degree, almost all of it sideways to the wheel, so that by 20° the contact has shifted 18 mm and by 60° about 50 mm (this is the migration of the touch point around the cambered rim described in the paper). A support that only turns about a fixed center, such as a lazy-susan bearing, still makes the tire slide sideways against friction. The front tire therefore sits on a *floating plate*: a small plywood plate resting on three ball-transfer units, which rolls freely in any direction and turns freely as well

(Fig. 4). The rear tire sits on a block of the same height so that the bicycle stays level (Fig. 3). What friction remains, mostly in the headset, is cancelled by approaching each angle from both directions and averaging.

Build the plate from a 200 mm square of 12 mm plywood with three 1-inch ball-transfer units (the steel-ball casters sold for conveyor tables, flange-mounted, about 30 mm tall) screwed to its underside in a triangle, balls down, so that the plate rolls on the floor on three steel balls. Glue two battens 40 mm apart on top, or cut a shallow groove, so that the tire sits centered and cannot roll off. The stack is about 45 mm high; make the rear block the same. The floor under the balls must be hard and smooth (a sheet of hardboard or a steel plate on a carpeted floor), and the plate needs about 6 cm of room to wander in every direction.

For the small-angle measurement alone, the two points at 10° and 20° that give the stiffness k, a thrust ball bearing is quicker: a 51206-size bearing (30 mm bore, 52 mm outer diameter, 16 mm thick) set on the floor under the tire. A 23 mm tire seats in the bore and cannot fall through, the bearing turns under it with almost no friction, and the sideways creep of the contact point is small enough, under 20 mm, that the tire stays on the upper race. Past 20° the contact walks off the bearing, so this option does not give the full curve or the flop angle. Raise the rear tire by the bearing's thickness.

The second difficulty is that the bicycle must be held exactly upright without the helper's hands supplying any steering torque. The helper holds the saddle only, with the top tube level (check with the level), and keeps the hands off the bars.

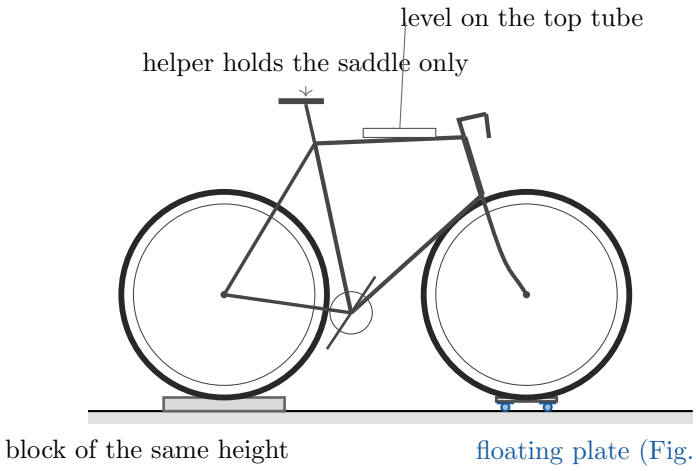


Figure 3: Torque measurement, side view. The front tire rests on the floating plate of Fig. 4 so that it can pivot and creep sideways without scrubbing, and the rear tire on a block of the same height. The bicycle is held upright by the saddle.

Front-wheel load N_f

Before the torque run, measure the weight on the front wheel (Fig. 5). Put the bathroom scale under the front tire and a block of the same height under the rear. For the unloaded bicycle, read the scale with the bicycle held upright by a fingertip on the saddle. Then have the person who will later sit on the bicycle mount it, in riding position with hands on the bars, while the helper steadies it, and read again. Use the reading that matches the condition of the torque run (usually unloaded, which is simpler). Multiply the mass by g to get N_f in newtons. A bare road bicycle gives about 40 N to 50 N; with a rider, 300 N to 400 N.

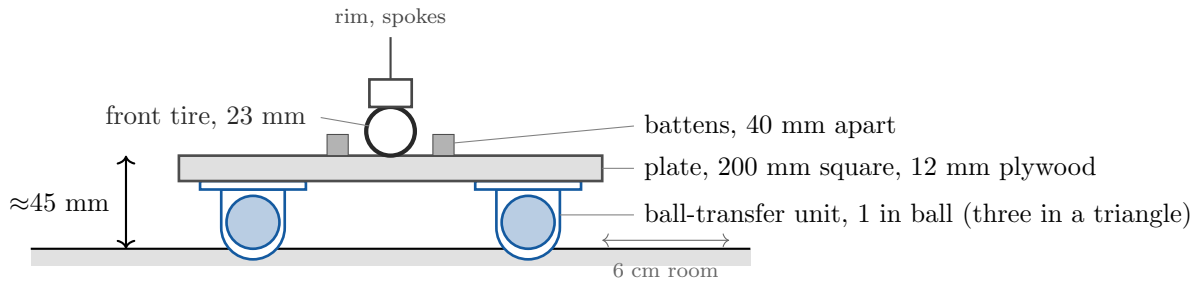


Figure 4: The floating plate. Three ball-transfer units screwed under a small plywood plate let it roll in any direction and turn, so the tire can pivot and its contact point can creep sideways, as the geometry requires, with rolling friction only. The rear block matches the height.

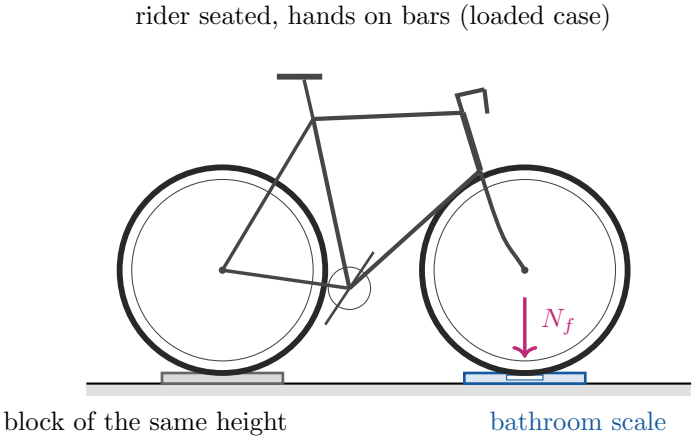


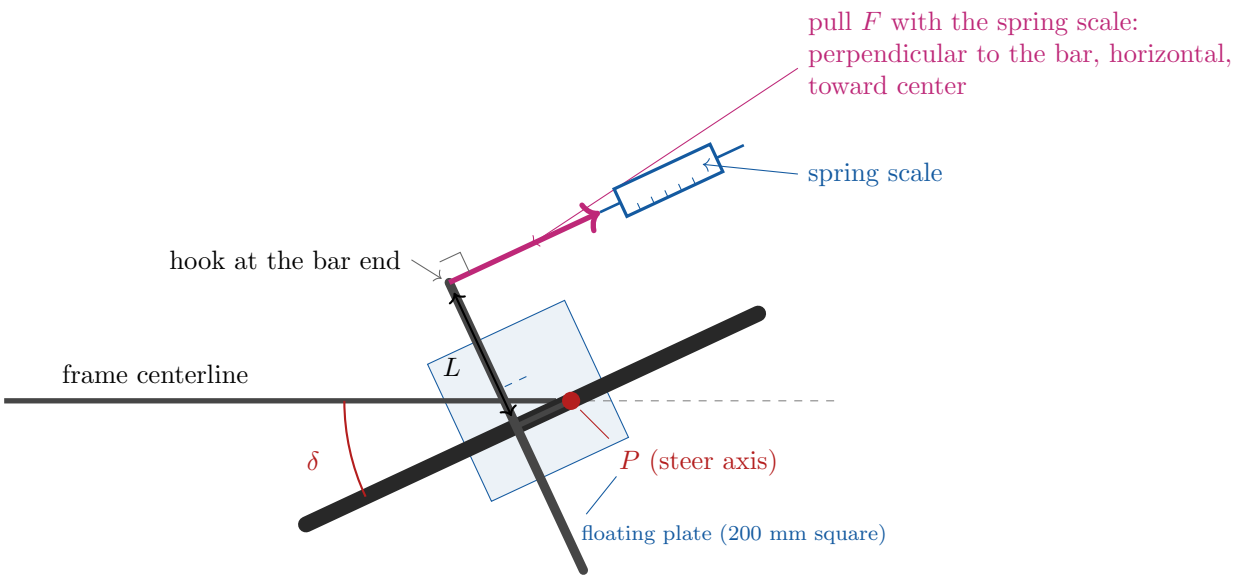
Figure 5: Measuring the front-wheel load N_f. The scale reads the share of the weight carried by the front tire; the rear tire sits on a block so that the bicycle is level.

Setup (Fig. 6)

1. Place the floating plate under the front tire, with the tire centered between the battens and the plate centered in its clear space, and the block under the rear tire. Check with the level that the top tube is as it was on the floor. Re-mark the floor and re-center the protractor on P; tape the protractor to the floor beside the plate, with the fork-crown pointer reaching over it, since the plate moves with the wheel.

2. Measure the lever arm L: the distance from the steer axis to the point on the handlebar where the scale's hook will pull, measured perpendicular to the steer axis. On a flat bar this is half the bar width minus a centimeter or so; on a drop bar hook onto the brake hood and measure from the axis to the hook. Record L.

3. Hook the 10 N spring scale to the end of the bar. Hold the scale so that its line of pull is perpendicular to the bar and horizontal, and parallel to the floor (Fig. 6). A pull that is not perpendicular reads low by the cosine of the error angle; 10° off costs 1.5%, so a visual check is enough. Pull from the outside of the turn, i.e., in the direction that would turn the bars back toward center.



Approach each δ twice: once letting the bars settle against the pull, once after pulling them 2°–3° past and easing back. Average the two readings.

Figure 6: Torque measurement, top view, bars turned left by δ. The spring scale pulls on the bar end along a line perpendicular to the bar and horizontal, in the direction that would return the bars to center. The torque is $T = FL$, with L the distance from the steer axis to the hook.

Procedure

1. Starting at $\delta = 10°$ to the left, let the bars settle against the pull of the scale and read the force F that just holds them. Then increase the pull until the bars move 2°–3° back toward center, ease off until they return to 10°, and read again. The first reading approached the angle from the far side, the second from the near side; record both on data sheet 3 and use their average. Their difference is twice the friction torque, which is worth recording too.
2. Repeat at 20°, 30°, …, up to the stop. Beyond 40° the torque falls, and past the flop angle δ_* (near 60°) the bars would rather return to δ_* than go on; there the scale must pull the other way, away from center, to hold the angle. Record the sign.
3. Repeat to the right.
4. If the loaded case is wanted, repeat with the rider seated. The torques are about seven times larger; use the 50 N scale, and have the rider keep hands off the bars.

What to expect

T rises almost linearly to a maximum near 35°, falls through zero at $\delta_* \approx 60°$, and reverses (Fig. 9b). For a bare road bicycle with $N_f \approx 45\,\mathrm{N}$ the maximum is about $0.25\,\mathrm{N\,m}$, which is a force of about $1.3\,\mathrm{N}$ on a 20 cm lever arm, so the 10 N scale must be read carefully; with a rider aboard ($N_f \approx 310\,\mathrm{N}$) the maximum is about $1.8\,\mathrm{N\,m}$. The slope at the origin, from the points at 10° and 20°, is k, and k/N_f is ζ_{eff} in meters. The spring scale feels the weight of the fork and bars as well as the flop, so this ζ_{eff} should come out several percent above the value from the drop lab; Sec. 4 says how to separate the two.

9 Runaway measurement

Idea

Released a few degrees off center, the bars accelerate away: the steer angle grows exponentially, $\delta(t) \propto e^{\sigma t}$, at a rate σ set by the flop stiffness and the inertia of the front assembly about the steer axis. Filming the release and fitting the growth gives σ; with the inertia I_δ measured separately, the stiffness follows.

Setup (Fig. 7)

1. Use the floating plate and block from the torque measurement, the floor protractor, and the fork-crown pointer.
2. Clamp the phone to the arm so that it looks straight down on the front end from about 1 m above, with the protractor and the full sweep of the bars in frame. Set the camera to its highest frame rate (240 fps on most phones). Put a strip of bright tape across the stem so that the bar's direction is easy to see in the video.
3. The helper holds the saddle with a light grip, enough to keep the bicycle from falling but not enough to stop the front end from sinking as the bars turn. This is the point of the exercise: the energy that drives the runaway is the sinking frame, and a rigid grip that holds the front end up removes it.

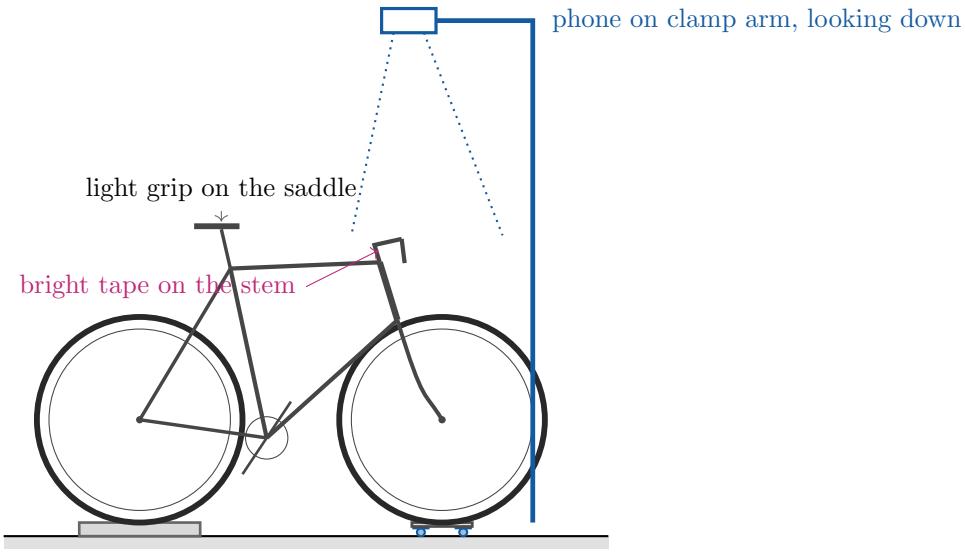


Figure 7: Runaway measurement. The phone films the front end from directly above at 240 fps. The floating plate from the torque measurement lets the tire pivot and creep freely, and the helper's grip on the saddle is light enough that the front end can sink as the bars run away.

Procedure

1. Set the bars at $\delta_0 = 3°$ to the left, hold them there by the bar end with one finger, start recording, and let go without pushing. The bars swing out and come to rest near δ_*; a release takes well under a second.

2. Repeat five times to the left and five to the right, and once or twice from 5° and 8° to see that the early growth rate does not depend on δ_0.

3. From the video, read δ every 4 frames (every 1/60 s) from the release until δ reaches about 25°, using the protractor in the frame. Enter the values on data sheet 4. Later frames are not used: the small-angle law holds only while $\delta \lesssim 25°$.

Steering inertia I_δ (Fig. 8)

The growth rate alone does not give ζ_{eff}; the inertia of the front assembly about the steer axis is needed too. Measure it as a torsion pendulum.

1. Remove the front assembly from the frame: loosen the stem and headset and withdraw the fork with the wheel and bars attached. (On most bicycles this is a five-minute job for someone who has done it; ask before starting.)

2. Clamp one end of a 1 m length of music wire (or braided line) to a beam or ceiling hook and the other end to the top of the steerer tube, with the wire along the steer axis, so that the assembly hangs with its steer axis vertical. It will hang tilted to one side because its center of mass is offset from the axis; that is expected.

3. Twist the assembly through a small angle and release. Time ten oscillations and record the period T_1.

4. Add a known inertia on the axis: slip a weight plate of mass m_p and inner and outer radii r_1, r_2 over the steerer, centered on the axis, and time ten oscillations again for T_2. The plate's inertia is $I_p = \frac{1}{2} m_p (r_1^2 + r_2^2)$, and $I_\delta = I_p \, T_1^2 / (T_2^2 - T_1^2)$. Measure r_1, r_2 with the caliper and m_p on the bathroom scale or a balance.

Expect I_δ of order $0.05\,\text{kg}\,\text{m}^2$ to $0.15\,\text{kg}\,\text{m}^2$ for a road bicycle; a 2.5 lb (1.13 kg) plate of 127 mm diameter adds $0.0025\,\text{kg}\,\text{m}^2$, so T_2 exceeds T_1 by only a few percent and the ten-oscillation timing must be done carefully. A heavier plate or a pair of slotted masses on a light bar across the axis gives a larger change. The paper's benchmark bicycle has $I_\delta = 0.30\,\text{kg}\,\text{m}^2$, but that is with a heavier front wheel than most bicycles carry.

What to expect

A plot of $\ln \delta$ against t is a straight line of slope σ once the bars are moving (the first few frames, while they are nearly at rest, bend the curve). For a bare road bicycle with $I_\delta \approx 0.1\,\text{kg}\,\text{m}^2$ expect σ of about $3\,\text{s}^{-1}$, so that the angle grows by a factor of e every 0.3 s; with a rider aboard, about $7\,\text{s}^{-1}$ (Fig. 9c). The release is over in well under a second, which is why the slow-motion camera is needed. Then $\sigma^2 I_\delta$ is the total gravitational stiffness, flop plus the hanging weight of the fork, as in the torque measurement.

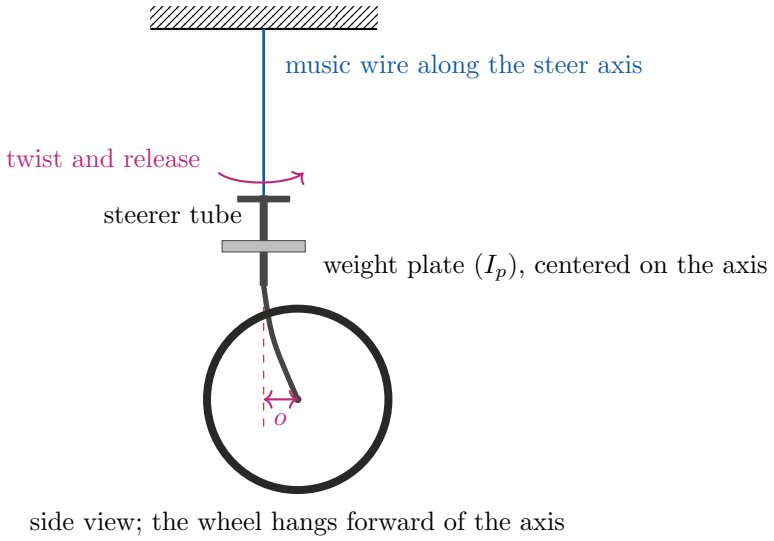


Figure 8: Measuring the steering inertia. The front assembly (fork, wheel, and bars) hangs from a wire along its steer axis and oscillates in torsion. Adding a weight plate of known inertia on the axis and timing again gives I_δ without knowing the wire's stiffness.

10 Expected results

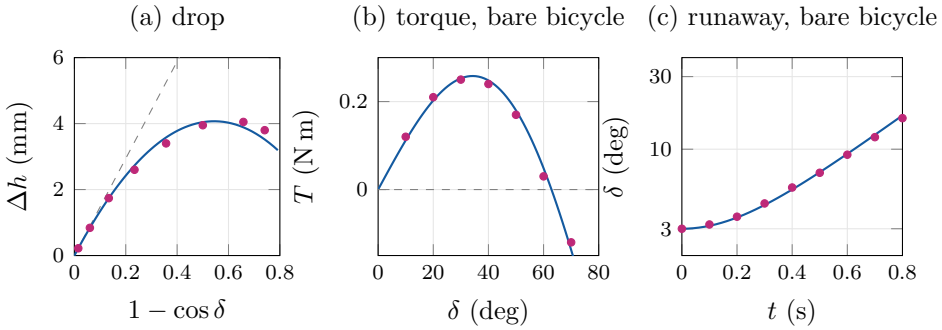


Figure 9: The shape of the data to expect, for the road geometry of Table 1. Curves are the predictions; dots are illustrative readings (not real data), showing roughly how the points should scatter. (a) The drop against $1 - \cos\delta$ with the small-angle line dashed. (b) The torque for the bare bicycle, $N_f = 45\,\mathrm{N}$; with a rider the curve is about seven times taller. (c) A release from 3° on a log scale; the slope of the straight part is σ.

11 Open questions

These are the things to settle before either measurement is given to students.

1. **The tire support.** Is the floating plate free enough that the friction torque, read as half the difference between the two approaches to each angle, is well below $k\delta$ at 10°? For a bare bicycle that is 0.12 N m; the plate's rolling friction on three steel balls under 45 N should be a few millinewton meters, but the battens and the tire's twist in the contact patch may add more. Try it with the bars at 10° and the scale reading from both sides.

2. **Holding the bicycle.** A helper at the saddle must not apply steering torque. Does the bottom-bracket stand of the drop lab do better? It takes most of the weight off the front wheel, so N_f falls to 10 N to 15 N for a bare bicycle; either use that value, or strap 4–5 kg to the frame near the head tube (not to the fork) and re-measure N_f.

3. **Small torques.** The bare-bicycle torque is a 1–2 N pull on a 10 N scale. Is a 5 N scale, or a digital force gauge, needed to get the 10° and 20° points to 5%? The loaded run is seven times larger but needs a rider who sits still with hands off the bars.

4. **The fork-weight term.** $g\, m_A u_A \sin\lambda$ must be separated from $N_f \zeta_{\text{eff}}$. Hanging the front assembly from its steerer (Fig. 8) gives u_A; is that accurate enough, or is the difference $k_{\text{tot}} - N_f \zeta_{\text{eff}}$ (with ζ_{eff} from the drop lab) the better route?

5. **Reading σ.** Is the log plot straight for long enough, between the first clear motion and 25°, to fix σ to a few percent at 240 fps? The predicted window is about half a second for a bare bicycle.

6. **Damping.** How much do headset friction and the plate's rolling resistance slow the runaway? Compare $\sigma^2 I_\delta$ with the static k_{tot}; a shortfall is the damping.

7. **The torsion pendulum.** With a 1.13 kg plate on a road fork, $T_2^2 - T_1^2$ may be only a few percent of T_1^2. Is a heavier added inertia needed to get I_δ to 5%?

Data sheet 3: the torque

Condition: [bare] / [rider] $N_f =$ _____ N $L =$ _____ m scale: _____

Sign: F is positive when the pull is toward center (holding the bars against the flop), negative when the pull is away from center.

δ (deg)	δ (rad)	F, far side (N)	F, near side (N)	$\bar{F}$ (N)	$T = \bar{F}L$ (N m)	friction $\frac{1}{2}L\lvert F_1 - F_2\rvert$
10 L	0.175					
20 L	0.349					
30 L	0.524					
40 L	0.698					
50 L	0.873					
60 L	1.047					
70 L	1.222					
10 R	0.175					
20 R	0.349					
30 R	0.524					
40 R	0.698					
50 R	0.873					
60 R	1.047					
70 R	1.222					

Slope through the origin (10°, 20°): $k_{\mathrm{tot}} =$ _____ N m/rad $k_{\mathrm{tot}}/N_f =$ _____ mm (upper bound on ζ_{eff})

Angle where F changes sign (observed δ_*): _____ deg Typical friction torque: _____ N m

Data sheet 4: the runaway

Frame rate: _____ fps frames per reading: _____ Δt per reading: _____ s condition: [bare] / [rider]

reading	Release 1, δ_0 = _____ δ (deg)	$\ln\delta$	Release 2, δ_0 = _____ δ	$\ln\delta$	Release 3, δ_0 = _____ δ	$\ln\delta$	Release 4, δ_0 = _____ δ	$\ln\delta$
0								
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								

Slope of $\ln\delta$ vs t: run 1 _____ run 2 _____ run 3 _____ run 4 _____ mean σ = _____ s^{-1} $\pm$ _____

Torsion pendulum. 10 periods, assembly alone: _____ s, _____ s, _____ s $\Rightarrow T_1$ = _____ s

10 periods, with plate: _____ s, _____ s, _____ s $\Rightarrow T_2$ = _____ s

Plate: m_p = _____ kg, r_1 = _____ mm, r_2 = _____ mm, $I_p = \frac{1}{2}m_p(r_1^2 + r_2^2)$ = _____ $\mathrm{kg\,m^2}$

$I_\delta = I_p T_1^2/(T_2^2 - T_1^2)$ = _____ $\mathrm{kg\,m^2}$

$k_{\mathrm{tot}} = \sigma^2 I_\delta$ = _____ N m/rad compare torque measurement: _____ N m/rad

Data sheet 5: summary

	Geometry	Drop lab	Torque	Runaway
ζ_{eff} (mm)				
δ_* (deg)			(sign change)	(rest angle)
k_{tot} (N m/rad)	$N_f \zeta_{\mathrm{eff}} =$			$\sigma^2 I_\delta =$

Which method gave the smallest uncertainty, and why?

Do the three values of ζ_{eff} agree within their uncertainties? If not, which systematic effect (tire squash, frame flex, friction, hanging-fork weight) is the likely cause, and in which direction does it push each measurement?