Bicycle wheel flop: the drop

A laboratory packet: theory, instructions, and data sheets

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Very preliminary draft. This packet has been designed but not yet run with students, and the numbers in it are predictions rather than results. The author would appreciate hearing from anyone who tries it, in whole or in part, or who finds an error: write to nick@aqualytics.eco. The current version is kept at http://www.aquahue.net.

Turn the handlebars of a bicycle and the front of the frame sinks. The sinking is small, a few millimeters, but it is the whole origin of what framebuilders call wheel flop: the steering torque that pulls the bars away from straight ahead is the weight on the front wheel times the rate at which the frame drops. This laboratory measures the drop as a function of steer angle with a dial indicator, with the bicycle held in an ordinary repair stand, and compares it with the geometric prediction, which has no free parameters once the bicycle has been measured. Part I gives the theory: the geometry, the assumptions of the model, stated as plainly as we can, and the drop law with its consequences. Part II gives the instructions, and the data sheets are at the end. The derivations are in the paper this packet accompanies[1] and are not repeated here. One three-hour session is enough for the geometry and two complete runs.

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[1] N. Tufillaro, "Bicycle wheel flop," submitted to *Eur. J. Phys.* (2026). Equation numbers in brackets, such as [paper, Eq. (22)], refer to that paper.

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Part I
Theory

1 The geometry

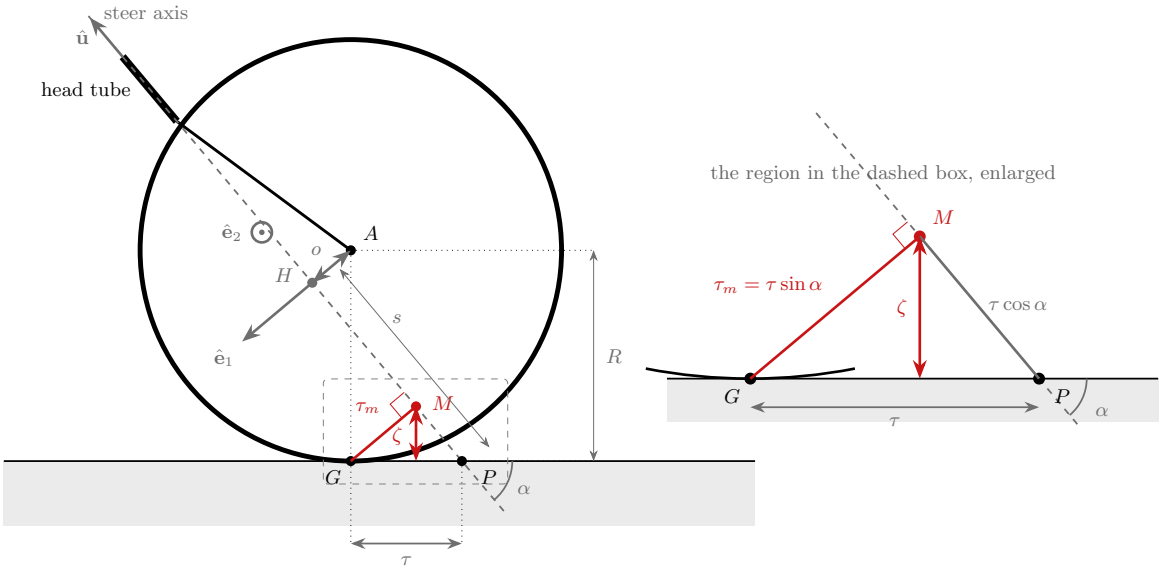


Figure 1: The front-end geometry in the bicycle's plane of symmetry, drawn with an exaggerated head angle and offset so that every length is legible (this is Fig. 3 of the paper). The steer axis meets the floor at P and makes the head angle α with it; the axle is A, a perpendicular distance o ahead of the axis; the tire touches the floor at G, directly below the axle. The ground trail τ is GP along the floor, the mechanical trail τ_m is the perpendicular from G to the axis, meeting it at M, and the flop factor ζ is the height of M above the floor. Grey marks the design parameters and red the quantities derived from them.

A bicycle is two rigid bodies joined by a hinge. The frame carries the rear wheel and the head tube; the front assembly (fork, front wheel, stem, and handlebars) turns inside the head tube about the steer axis, through the steer angle δ. The axis leans back at the top and strikes the floor at P, a little ahead

of the point G where the tire touches; the angle it makes with the floor is the head angle α, about 73° on a road bicycle. We use α throughout and never its complement, the tilt of the axis from the vertical, which the bicycle-dynamics literature prefers; the two are a swap of sines and cosines, and keeping one of them avoids a good deal of confusion. Figure 1 shows the rest, and Table 1 lists each symbol with how it is obtained.

Table 1: Geometric symbols. "Measured" means read directly in Sec. 9; "computed" means obtained from the measured ones by the equation given. The last column is the bicycle of the worked example in Sec. 5.

Symbol	Meaning	How obtained	Example
α	head angle, steer axis to the floor	measured, two ways (Sec. 9)	71°
R	radius of the inflated wheel, floor to axle	measured, two ways	345 mm
ρ	half the inflated tire width	measured, caliper	11.5 mm
o	fork offset, axle A to steer axis, perpendicular	measured, two rules	40 mm
τ	ground trail, G to P along the floor	computed, Eq. (1); checked on the floor	76.5 mm
τ_m	mechanical trail, G to the axis, perpendicular	computed, Eq. (2)	72.3 mm
ζ	flop factor, knife-edge wheel	computed, Eq. (3)	23.5 mm
ζ_{eff}	flop factor with the tire	computed, Eq. (4)	22.3 mm
δ	steer angle from straight ahead, about the steer axis	measured, protractor on the head tube (Sec. 10)	0–75°
δ_g	angle of the wheel's line on the floor	computed, Eq. (9); can be measured instead	a little less than δ
γ	camber (lean) angle of the front wheel	computed, Eq. (6)	0–18°
δ_*	flop angle, where the drop is largest	computed, Eq. (8); observed	69.5°

The lengths that follow from the measured ones are

$$\tau = \frac{R\cos\alpha - o}{\sin\alpha} \qquad \text{[paper, Eq. (8)]} \tag{1}$$

$$\tau_m = \tau\sin\alpha = R\cos\alpha - o \qquad \text{[paper, Eq. (9)]} \tag{2}$$

$$\zeta = \tau_m\cos\alpha = \cos\alpha\,(R\cos\alpha - o) = \tau\sin\alpha\cos\alpha \qquad \text{[paper, Eq. (15)]} \tag{3}$$

$$\zeta_{\text{eff}} = \cos\alpha\,\big[(R-\rho)\cos\alpha - o\big]. \qquad \text{[paper, Eq. (24)]} \tag{4}$$

The trail is the shadow the mechanical trail casts on the floor; the mechanical trail is the lever arm through which the floor acts on the steering; and the flop factor is that lever arm scaled by the cosine of the head angle, which is the height of the point M in Fig. 1. Equation (4) is Eq. (3) with R replaced by $R-\rho$: a fat tire flops like a thin wheel whose radius reaches only to the middle of the tread. The experiment measures ζ_{eff}, and the geometric value from Eq. (4) is the prediction it is compared with.

2 The model and its assumptions

The drop law of Sec. 3 is a statement about geometry, not about forces, and it pays to be explicit about what is assumed, because the assumptions are exactly the things the laboratory setup has to enforce.

Two rigid bodies and a hinge. The bicycle is treated as two rigid bodies, the frame (with the rear wheel, which plays no part) and the front assembly, hinged along the steer axis. The steer angle δ is

the one coordinate of that hinge. The wheel is a rigid torus, a tube of radius ρ whose centerline is a circle of radius $R - \rho$ about the axle. Nothing bends and nothing squashes.

The frame is held. A rigid body has six degrees of freedom. The model fixes five of them for the frame: it may not roll (lean sideways), yaw (turn about a vertical axis), or pitch (nod about a horizontal axis), and it may not move horizontally. It may move vertically, and that is the only freedom it has. In the experiment (Part II), roll and yaw are removed by the stand's cradle, horizontal motion by the stand's fixed position, and pitch by nothing at all: a bottom-bracket stand leaves the frame free to pitch about the stand, and that is the one place the laboratory departs from the model (Sec. 4). What the model calls vertical translation the laboratory realizes as a small pitch, and the lever factor x_G/x_{cap} is the bookkeeping that connects the two.

The steer axis has a fixed direction. Because the frame's attitude is fixed, the steer axis is a line of fixed direction (it moves up and down with the frame, and nothing else), making the angle α with the floor in the bicycle's plane of symmetry. Turning the bars by δ therefore rotates the whole front assembly rigidly about that line, and everything about the front wheel follows from this one rotation [paper, Eq. (13)]. The axle swings around the axis on a circle of radius o, dropping by $o\cos\alpha(1-\cos\delta)$ relative to the axis [paper, Eq. (16)], and the wheel plane, which was vertical at $\delta = 0$, leans by the camber angle γ with $\sin\gamma = \cos\alpha\sin\delta$ [paper, Eq. (18)]. The axle is horizontal only at $\delta = 0$; at $\delta = 30^\circ$ it is tilted by 9.4° for the example geometry. Nothing is adjusted to keep the frame upright; the frame is held upright, and the wheel's lean is the consequence.

The floor supplies one constraint. The floor is a rigid horizontal plane, and the only condition it imposes is that the lowest point of the tire lie on it. Here is the counting that makes the drop a certainty rather than something to be solved for. Once δ is chosen, the front wheel's position relative to the frame is completely determined, since the hinge has one coordinate and it has been set, so the height of the wheel's lowest point relative to the frame is a definite function of δ. The frame has exactly one freedom left, its height, and the floor's one condition uses it up. The frame *must* sit at the height that puts the tire's lowest point on the floor, and $\Delta h(\delta)$ is the amount that height differs from its value at $\delta = 0$. The frame drops because, with the wheel leaned and the axle swung, the tire's lowest point is closer to the frame than it was, and the floor pushes the frame down to meet it.

The lowest point of a leaned torus is easy to find. Let $\mathbf{c}$ be the center of the wheel and $\mathbf{n}$ the unit normal to the wheel plane (the axle direction) after the rotation. The direction in the wheel plane that points most steeply up is $\mathbf{w} = (\hat{\mathbf{z}} - (\mathbf{n}\cdot\hat{\mathbf{z}})\mathbf{n})/|\hat{\mathbf{z}} - (\mathbf{n}\cdot\hat{\mathbf{z}})\mathbf{n}|$, and the contact point is

$$\mathbf{G}(\delta) = \mathbf{c}(\delta) - (R-\rho)\,\mathbf{w}(\delta) - \rho\,\hat{\mathbf{z}}. \tag{5}$$

The lowest point lies $(R-\rho)\cos\gamma + \rho$ below the axle, which is where the first term of Eq. (6) comes from.

The contact point is not stationary. Nothing above says that the tire touches the floor at a fixed spot; it touches wherever its lowest point happens to be, and Eq. (5) says that spot moves. For the example geometry it walks about 1.2 mm per degree of steer at first, sideways toward the outside of the lean (to the rider's right when the bars go left), and increasingly forward at large angles: 24 mm from its starting point at 20° and 61 mm at 60°, returning to the plane of symmetry at the flop angle. This has no effect on the drop, which depends only on the lowest point's height, but it matters for how the tire is supported. A wheel cannot roll sideways, so on a fixed floor a stationary bicycle's tire must scrub across the surface as the bars turn; the lift-turn-set-down procedure of Sec. 11 avoids the scrub by placing the tire fresh at each angle.

Rolling is irrelevant. The wheel is a surface of revolution about its axle. Spinning it about the axle moves no point of the surface anywhere new, so whether the wheel rolls, slides, or is locked makes no difference to where its lowest point is. The model does not assume rolling, and does not assume that the wheel is prevented from rolling; it does not need to say anything about it.

Friction is not in the drop, but it is in the torque. Because $\Delta h(\delta)$ is fixed by geometry alone, friction at the contact cannot change it: friction can resist the motion of the bars, but it cannot put the frame anywhere other than the one height the constraints allow. (The exception is a flexible tire, which friction can distort by a fraction of a millimeter; see Sec. 6.) Friction does enter the torque. The flop torque of the paper is the conservative torque $T = -dU/d\delta$ with $U = -N_f\,\Delta h$ [paper, Eq. (25)], which is the whole steering torque only if the floor does no work as the contact point moves, that is, if the tire moves without sliding friction. That is true for a rolling bicycle, where the sideways drift of the contact point is taken up by the rolling path curving, and it is why the torque measurement, a separate laboratory still in development, must support the tire on something that lets the contact point move freely.

Quasi-static. The bars are turned slowly, so the drop is a sequence of equilibrium positions and no inertia enters. The runaway measurement, also in development, is the dynamic version, in which the same potential drives the bars away from center at a rate set by the steering inertia.

What is left out, and what it costs. Four things are left out. Tire squash and fork flex are the first two; we expect both to make the measured drop a few percent smaller than Eq. (6), and Question 6 asks you to argue which way each goes. The third is the frame's pitch on the stand, which is treated in Sec. 4: up to 10% at δ_* and nothing at small angles. The fourth is everything that produces a torque without changing a height, namely headset friction, which resists the turning, and the weight of the front assembly, which adds a small torque in the torque laboratory; neither touches the drop, which is pure geometry. Table 2 collects the assumptions and says what in the laboratory enforces each one.

Table 2: The model's assumptions, how the laboratory enforces them, and what happens if they fail.

Assumption	Enforced by	If violated
Rigid frame and fork; rigid torus tire	stiff bicycle, tire at rated pressure	drop expected to read low by a few percent (squash, flex)
Frame does not roll	stand cradle; bubble level across the top tube	odd-in-δ term; cancels in the L/R average
Frame does not yaw or move horizontally	stand and platform fixed on tape marks	lever arms change; readings do not repeat
Frame does not pitch; head angle fixed	*not* enforced by a bottom-bracket stand: frame pitches by $\Delta h/x_G$	dial reads x_{cap}/x_G of the drop (corrected by the lever factor); head angle steepens by under a degree (up to 10% at δ_*)
Frame attitude = on-road attitude	angle gauge on the head tube reads α in the stand	ζ_{eff} off by 13% per degree
Floor is a rigid horizontal plane	hard smooth platform, shimmed level	tire sinks into a soft surface; drop reads low
Tire's lowest point is on the floor; contact point free to be anywhere	lift-turn-set-down, so the tire is placed, not scrubbed	scrub winds up the sidewall; sub-mm offsets, hysteresis between L and R approaches
Quasi-static	bars turned slowly, read at rest	frame bounces; dial unsettled

3 The drop

With the frame held at fixed attitude, the front of the frame sinks by

$$\Delta h(\delta) = (R-\rho)\,(1-\cos\gamma) - o\cos\alpha\,(1-\cos\delta), \qquad \sin\gamma = \cos\alpha\,\sin\delta, \qquad \text{[paper, Eq. (22, 24)]} \quad (6)$$

where γ is the camber angle of the wheel, the angle by which the wheel plane leans from vertical when the bars are turned about the tilted steer axis. The first term is the rise of the lowest point of a leaned wheel toward its axle; the second is the fall of the axle as it swings around the axis on the offset o. Nothing has been linearized: Eq. (6) holds at any steer angle for a rigid torus on a rigid front end that sinks without changing its head angle. For small steer angles it reduces to the framebuilders' rule

$$\Delta h \simeq \zeta_{\text{eff}}\,(1-\cos\delta) \simeq \tfrac{1}{2}\,\zeta_{\text{eff}}\,\delta^2, \qquad \text{[paper, Eq. (23)]} \quad (7)$$

and the two first differ at fourth order, $\Delta h_{\text{rule}} - \Delta h = (R-\rho)\sin^2\alpha\cos^2\alpha\ \delta^4/8 + \ldots$, so the rule overestimates the drop by about 1% at 10°, 11% at 30°, 27% at 45°, and 59% at 60° for the example geometry. The exact drop reaches its maximum at the flop angle

$$\cos\delta_* = \frac{\beta\sin\alpha}{\sqrt{1-\beta^2\cos^2\alpha}}, \qquad \beta = \frac{o}{(R-\rho)\cos\alpha}, \qquad \text{[paper, Eq. (27)]} \quad (8)$$

which exists whenever $\beta < 1$, that is, whenever $\zeta_{\text{eff}} > 0$. At δ_* the tire's contact point lies in the plane of symmetry of the bicycle, directly beneath the steer axis; past it the front end rises again.

The angle on the floor. The steer angle δ is a rotation about the tilted steer axis. The angle through which the wheel's line on the floor has turned from straight ahead, δ_g, is a little smaller, because part of a rotation about a tilted axis goes into leaning the wheel rather than turning it:

$$\tan\delta_g = \sin\alpha\,\tan\delta, \qquad \text{[paper, Eq. (20)]} \quad (9)$$

which for the example geometry makes δ_g half a degree less than δ at 10° and 1.6° less at 45°. If the angle you measure is δ_g rather than δ (see Sec. 10), the drop can be written in it directly. With $D = \sqrt{1-\cos^2\alpha\cos^2\delta_g}$,

$$\cos\delta = \frac{\sin\alpha\cos\delta_g}{D}, \qquad \sin\gamma = \frac{\cos\alpha\sin\delta_g}{D}, \qquad \text{[paper, Eq. (21)]} \quad (10)$$

and Eq. (6) with these substituted is the drop as a function of δ_g; the two forms describe the same position of the wheel and give the same number.

What ζ_{eff} means. It is a length: the curvature of the drop curve at $\delta = 0$, or the drop the front end would make if the bars could turn through 90° by the rule (7). It matters because a drop under load is a torque. With N_f the weight on the front wheel, the steering torque pulling the bars away from center is $T \simeq N_f\zeta_{\text{eff}}\,\delta$ for small angles, a negative spring of stiffness $k = N_f\zeta_{\text{eff}}$ [paper, Eq. (26)]. That torque is what a rider feels as wheel flop, and the drop measured here is nearly the whole of its origin; the weight of the front assembly adds a little.

4 The stand: lever factor and pitch

In the arrangement of Part II the bicycle sits in a stand under the bottom bracket, so the frame cannot translate; it pitches about the stand. The tire's contact point is a horizontal distance x_G ahead of the

stand and the dial's tip, on the stem cap, a distance x_{cap}. The drop at the tire is Δh, the frame pitches by $\varepsilon = \Delta h / x_G$, and the dial reads

$$d = \varepsilon\, x_{\text{cap}} = \Delta h\, \frac{x_{\text{cap}}}{x_G}, \qquad \text{so} \qquad \Delta h = d\, \frac{x_G}{x_{\text{cap}}}. \qquad \text{[paper, Eq. (33)]} \tag{11}$$

For $x_G = 0.58\,\text{m}$ and $x_{\text{cap}} = 0.38\,\text{m}$ the factor is 1.5. Both lengths are measured with the tape at $\delta = 0$; the ratio is what matters, and a 5 mm error in either is under 1.5%.

The pitch itself is small, $\varepsilon \simeq 7\,\text{mm}/0.58\,\text{m} \simeq 0.7^\circ$ at δ_* for the example bicycle, but it steepens the head angle by ε as the front end sinks, and Eq. (6) assumes a fixed head angle. Since ζ_{eff} changes by about 13% per degree (Sec. 6), the drop at δ_* comes out roughly 10% smaller than Eq. (6) with the $\delta = 0$ head angle; at 30° the effect is about 3%, and it vanishes at small angles, so the slope that gives ζ_{eff} is unaffected. (On a bicycle standing on both wheels the pivot is the rear contact, x_G is the wheelbase, and the effect is about half as large; the paper quotes 5% for its road geometry.) A student who wants to remove it can evaluate Eq. (6) with α replaced by $\alpha + \varepsilon(\delta)$ at each angle; it is one extra column. Table 3 summarizes what is measured and what is extracted from it.

Table 3: What is measured and what is extracted.

Symbol	Meaning	In the experiment
δ	steer angle	read on the head-tube protractor at each step
d	dial reading at the stem cap	zeroed at $\delta = 0$; left and right runs averaged
x_G, x_{cap}	horizontal distances from the stand's pivot to the tire contact and to the dial tip	tape measure, at $\delta = 0$
Δh	drop of the frame at the front wheel	$d\, x_G / x_{\text{cap}}$, Eq. (11)
ζ_{eff}	flop factor	slope of Δh against $1 - \cos\delta$ for $\delta \le 20^\circ$, or the fit of Eq. (6)
δ_*	flop angle	the δ at which Δh is largest; compare with Eq. (8)
ε	pitch of the frame	angle-gauge creep during the run; should equal $\Delta h / x_G$

5 Worked example

For a bicycle with $\alpha = 71^\circ$, $R = 345\,\text{mm}$ (measured from the floor to the axle, rim plus inflated tire), $\rho = 11.5\,\text{mm}$ (a 23 mm tubular tire), and $o = 40\,\text{mm}$, so that $R - \rho = 333.5\,\text{mm}$ and $\cos 71^\circ = 0.3256$:

$$\begin{aligned}
\tau &= \frac{0.345\cos 71^\circ - 0.040}{\sin 71^\circ} = 76.5\,\text{mm}, \qquad \tau_m = \tau \sin 71^\circ = 72.3\,\text{mm},\\
\zeta &= \cos 71^\circ\,(0.345\cos 71^\circ - 0.040) = 23.5\,\text{mm},\\
\zeta_{\text{eff}} &= \cos 71^\circ\,(0.3335\cos 71^\circ - 0.040) = 22.3\,\text{mm},\\
\beta &= \frac{0.040}{0.3335\cos 71^\circ} = 0.368, \quad \cos\delta_* = \frac{0.368\sin 71^\circ}{\sqrt{1 - 0.368^2\cos^2 71^\circ}} = 0.351,\\
\delta_* &= 69.5^\circ, \qquad \Delta h(10^\circ) = 0.34\,\text{mm}, \qquad \Delta h(20^\circ) = 1.29\,\text{mm}, \qquad \Delta h(\delta_*) = 7.42\,\text{mm},\\
\Delta h(30^\circ) &= 2.70\,\text{mm} \quad (\text{rule } \zeta_{\text{eff}}(1 - \cos\delta)\text{: } 2.99\,\text{mm}).
\end{aligned}$$

With $x_G = 0.58\,\text{m}$ and $x_{\text{cap}} = 0.38\,\text{m}$ the dial reads $0.66\,\Delta h$: 0.22 mm at 10°, 1.77 mm at 30°, and 4.9 mm at the flop angle by Eq. (6) at the $\delta = 0$ head angle, where the frame has pitched by $7.42/580 =$

12.8 mrad = 0.73° and the angle gauge on the head tube reads 71.7°. Allowing for that pitch as in Sec. 4 trims the drop at the flop angle to about 6.6 mm and the dial reading to about 4.4 mm. With a rider's 310 N on the front wheel this geometry gives a steering stiffness $k = N_f \zeta_{\text{eff}} = 6.9\,\text{N m/rad}$, which is what the torque laboratory, when it is ready, will measure with a spring scale.

6 Uncertainties worth tracking

- α is the most sensitive input. From Eq. (4), $d\zeta_{\text{eff}}/d\alpha = -\sin\alpha\,[2(R-\rho)\cos\alpha - o]$, which is 2.9 mm per degree for the example, or about 13% of ζ_{eff} per degree of head angle. Read it to 0.1°, by both methods of Sec. 9, and set the attitude in the stand to the same 0.1°; a platform left 10 mm too low pitches the frame by 1° and takes 13% off the measured ζ_{eff}.
- o is the hardest length to read and enters ζ_{eff} directly: $\Delta\zeta_{\text{eff}} = \cos\alpha\,\Delta o$, so a 2 mm error in o is 0.65 mm, or 3%, in ζ_{eff}.
- R enters as $\cos^2\alpha\,\Delta R$: a 5 mm error in R is 0.5 mm, or 2.4%. Measure the inflated wheel, not the rim.
- δ matters more than one might think, because the slope is taken against $1-\cos\delta$, and $d(1-\cos\delta)/(1-\cos\delta) = \cot(\delta/2)\,d\delta$: a 1° error is 10% of $1-\cos\delta$ at 20° and 20% at 10°. Read the disc on the head tube, which measures δ in its own plane, to the nearest half degree or better; the errors are random from point to point and the slope fit averages them down, but they are the largest single source of scatter.
- The lever factor x_G/x_{cap} to 1% needs each length to about 4 mm, easy with a tape. Be sure x_G is to the contact point, directly below the axle, not to the front of the tire.
- The dial's own repeatability is the floor: with the base rigid, readings should return to zero within 0.02 mm after a lift. Anything worse is the base moving or the tire not settling.

Part II
Experiment

7 What you will need

Figure 2 shows the arrangement; most of it a physics department already owns.

- A bicycle. A road or touring bicycle with narrow tires gives the cleanest numbers; a mountain bike works but has more tire squash. Inflate the tires to their rated pressure before starting.
- A bottom-bracket stand: the kind of race or display stand that cradles the bicycle under the bottom bracket, or plugs into a hollow crank spindle, and lifts the rear wheel a few centimeters off the floor. The bicycle must sit in it upright and must be able to pitch about the stand by a degree or so.
- A platform for the front wheel: a square of 12 mm plywood or hardboard, roughly 300 mm on a side, plus a few thin shims (hardboard offcuts, sheets of card) to adjust its height.
- Geometry: a digital angle gauge with a magnetic base (0.1°), a 1 m rule or straightedge, a 300 mm rule, a caliper, a tape measure, a small bubble level, and a piece of chalk.
- Drop: a dial indicator (25 mm travel, 0.01 mm) on a magnetic base with an articulating arm, and a rigid stand or a heavy steel plate for the base; the printed steer-angle protractor, one sheet (`flop_protractor_head.pdf`), and a sewing needle or a strip of tape with a fine line on it, for a pointer on the stem; painter's tape, scissors, a sheet of paper, and a floor pump with a gauge.

No helper is needed, since the stand holds the bicycle.

8 Safety and setup

Keep fingers clear of the gap between the fork and the frame when the bars swing; once the bars are past 20° or so they want to keep going. Make sure the stand cannot tip: a bicycle on a bottom-bracket stand is balanced on the stand and the front tire, so it rolls over easily if pushed sideways. Work on a hard, level floor, and put the platform and the stand on tape marks so that everything can be returned to the same spot.

9 Geometry of the front end

The prediction needs three lengths and one angle, R, ρ, o, and α, and the trail is measured as a check on them, so measure these first and record them on data sheet 1. Table 1 in Part I defines each symbol and Fig. 3 shows where the measurements are made. Measure them with the bicycle standing on both wheels on the floor, before it goes into the stand. Each quantity is measured two ways where that is practical. The second way is not a check in the usual sense, since the two methods often measure slightly different things; it is there so that you can decide, with numbers in hand, which to trust, and the questions of Sec. 13 ask you to.

Head angle α. Measure it two ways. First, with the angle gauge: zero it on the floor, stick it to the side of the head tube, read the angle, and repeat on the other side of the tube; average the two. (The gauge reads the angle from the horizontal, which is how α is defined. If your gauge reads from the vertical, subtract its reading from 90°.) Second, with the 1 m rule and a triangle: lay the rule along

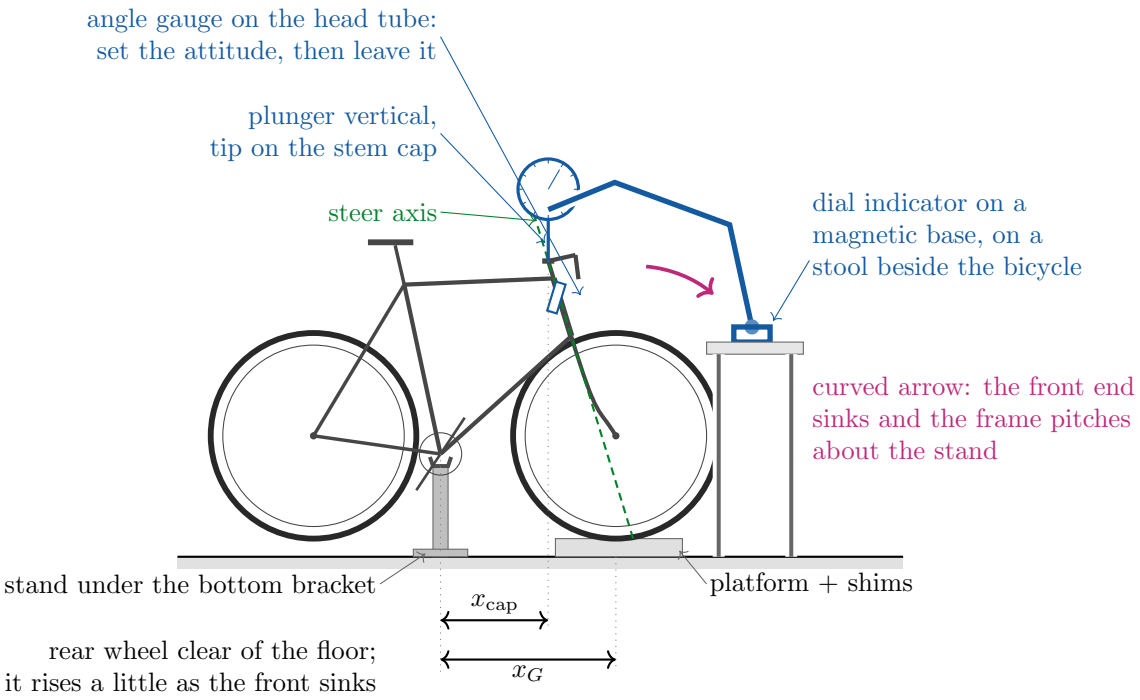


Figure 2: The arrangement. The bicycle sits in a bottom-bracket stand with the rear wheel off the floor and the front tire on a platform shimmed so that the frame has its on-road attitude. The dial indicator stands beside the bicycle, not on it, with its plunger on the stem cap. As the bars turn the front end sinks and the frame pitches about the stand; the dial reads that sinking at the stem cap, a distance x_{cap} ahead of the stand, and the lever factor x_G/x_{cap} converts it to the drop at the tire. The lift of the rear wheel is exaggerated. The steer axis is dashed green, as in the paper.

the head tube so that its edge extends down to the floor, mark where the edge meets the floor, choose a point on the edge well up the rule, and measure that point's height h above the floor with the 300 mm rule held vertical and its horizontal distance b from the floor mark; then $\tan\alpha = h/b$, or, reading the distance L along the rule instead of b, $\sin\alpha = h/L$. Record both results and their difference. Expect 70°–74°. This is the most sensitive number in the experiment: near 71° the flop factor changes by about 13% per degree, so it is worth asking which of the two methods you trust to 0.1°, and why. The attitude of the bicycle in the stand will later be adjusted until the gauge repeats this reading (Sec. 11, Setup, step 2), so the number recorded here is the one the whole experiment is referred to.

Wheel radius R. Again two ways. Directly: with the bicycle upright on the floor and the tire inflated, measure from the floor to the center of the front axle with the rule held vertical. By rolling: put a chalk mark on the tire at the contact point, roll the bicycle forward in a straight line until the mark is back at the floor, measure the distance between the two floor marks, and divide by 2π. Take two readings each way. R is the radius of the inflated tire, rim and tire together, not the rim. The two methods do not measure quite the same thing: the rolled distance is set by the loaded tire, flattened at the contact, while the axle height is the loaded radius at the one point that matters for the drop.

Tire half-width ρ. Measure the inflated tire's width with the caliper at the widest point of the casing; ρ is half of it. A clincher marked "28" usually measures 27 mm to 30 mm; a 23 mm tubular is close to its marking. As a second estimate, measure the width of the tire's footprint on the floor (dust

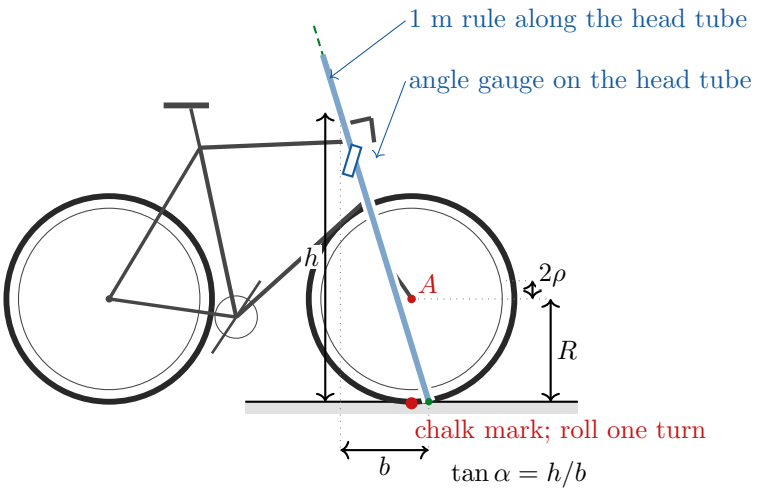


Figure 3: Where the measurements are made. α is read with the magnetic angle gauge stuck to the head tube and, independently, from the triangle formed by the 1 m rule laid along the head tube down to the floor: a point on the rule at height h and horizontal distance b from the rule's foot gives $\tan\alpha = h/b$. R is the floor-to-axle height with the tire inflated, and also the distance rolled in one turn of the chalk mark divided by 2π; 2ρ is the inflated tire width. The offset o and the trail τ are too small to draw at this scale; Fig. 1 shows them exaggerated.

the floor, or roll over a sheet of paper) and compare: the footprint is narrower than the casing, and the model's torus of tube radius ρ is an idealization of a tire that is neither round nor rigid.

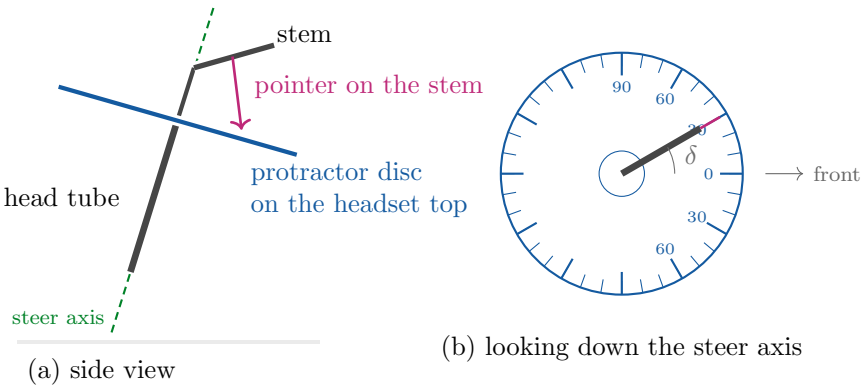
Fork offset o. Lay the 1 m rule along the head tube, touching it on the side, so that the rule's edge is parallel to the steer axis and extends down past the axle. Measure the perpendicular distance from the center of the axle to the rule's edge with the 300 mm rule. Repeat on the other side and average. Typical values are 40 mm to 55 mm. A second route is through the trail: measure τ on the floor as in the next paragraph and solve Eq. (1) for $o = R\cos\alpha - \tau\sin\alpha$. Since o is then the small difference of two larger lengths, think about how an error in α propagates into it by this route before you decide which value to trust.

Trail τ (check). Compute $\tau = (R\cos\alpha - o)/\sin\alpha$ from the numbers above. Then check it directly: with the front wheel straight, lay the 1 m rule along the head tube again, this time extending it down to the floor, and mark where its edge meets the floor; that is P. Mark where the tire touches the floor, directly below the axle; that is G. The distance GP is the trail. The two values should agree within a few millimeters; if they do not, re-measure o, which is the one most easily read wrong.

10 Measuring the steer angle δ

The steer angle is the angle through which the front assembly has been turned about the steer axis from straight ahead, and the cleanest way to read it is in the plane in which it is defined, the plane perpendicular to the steer axis. The scale is the printed protractor disc (`flop_protractor_head.pdf`; Fig. 4). Cut out its center hole and the slit, slip it around the steerer just below the stem, and tape it to the top of the headset so that it lies flat against the top of the head tube, with its 0 line pointing straight ahead along the top tube. Because the disc sits on the head tube, it is fixed to the frame and perpendicular to the axis; the stem turns above it. The pointer is a needle taped to the underside of

the stem, or a strip of tape on the stem with a fine line drawn on it, pointing down at the scale a millimeter or two above the paper. With the bars straight the pointer should sit on 0; if it does not, turn the disc, not the bars, until it does.



(a) side view

(b) looking down the steer axis

Figure 4: Reading δ. (a) The paper protractor lies on the top of the head tube, perpendicular to the steer axis, and a pointer fixed to the stem sweeps over it. (b) Seen along the axis, the pointer reads the steer angle directly. Turning the bars to the left is positive.

Other ways to read the angle. There are at least three, and it is worth thinking about why they give different numbers.

1. A protractor on the floor, centered on P, with a pointer taped to the fork. This is the obvious arrangement, and it was the one first planned for this laboratory. But a pointer fixed to the fork turns about the tilted steer axis, and its shadow on the floor does not turn through δ. If the pointer is mounted perpendicular to the steerer, its floor reading δ_p obeys $\tan\delta_p = \tan\delta/\sin\alpha$, larger than δ by 1.6° at 45° for the example geometry; if it is mounted horizontal the relation is messier and the reading falls short of δ by nearly 3° at 45°. Either can be converted, but the conversion has to be done.

2. A straightedge laid on the floor along the tire's footprint, read against the same floor protractor. This measures δ_g, the angle of the wheel's line on the floor, Eq. (9), and the drop can be written in δ_g directly, Eq. (10). It is the hardest of the three to read to a degree, since a footprint is a few centimeters long.

3. The disc on the head tube, above. It reads δ itself, with no conversion, because it lies in the plane of the rotation.

Which method is best depends on what you want the angle for. The theory is written in δ; the floor is where the bicycle meets the road. Before you take data, decide which angle you are measuring, write it on the data sheet, and consider the questions in Sec. 13.

One caution is in order. The angle you want is the angle of the front wheel, not of the handlebars; on a rigid bicycle they are the same, but a loose headset or a bent stem makes them differ. Check once, with the bars straight, that the wheel is straight too.

11 The drop measurement

Idea

As the handlebars turn through δ the front of the frame sinks by $\Delta h(\delta)$. The small-angle rule says $\Delta h \simeq \zeta_{\text{eff}}(1 - \cos\delta)$, so a plot of Δh against $1 - \cos\delta$ is a straight line through the origin with slope ζ_{eff}, the flop factor. The exact law bends below that line at large angles and turns around at the flop angle δ_*, near 70° for the example bicycle: past δ_* the front end rises again.

The bicycle is held by a stand under the bottom bracket, so the frame can do only one thing as the bars turn, pitch nose-down about the stand. The dial indicator reads that pitch at the stem cap. Because the stem cap is nearer the stand than the tire is, it moves less than the tire's contact point does, by the lever factor x_G/x_{cap} (Fig. 2); Sec. 4 gives the one-line correction.

Setup (Fig. 2)

1. **Stand and platform.** Put the bicycle in the stand with the rear wheel a few centimeters off the floor and the front tire on the platform, bars straight. Check with the bubble level across the top tube that the frame is not leaning; if the stand lets it lean, shim the stand.

2. **Attitude.** The frame must have the same attitude it has on the road, because the attitude sets the head angle. Stick the angle gauge to the head tube, where it was for the geometry measurement, and add or remove shims under the platform until it reads the α recorded on data sheet 1, to 0.1°. Leave the gauge on the head tube for the rest of the run, but do not adjust the shims again: the reading will creep up by a few tenths of a degree as the bars turn, and that creep is the measurement.

3. **Lever arms.** With the tape, measure the horizontal distance from the center of the bottom bracket to the front tire's contact point, x_G, and from the bottom bracket to the point on the stem cap where the dial's tip will rest, x_{cap}. Record both on data sheet 1, under "In the stand"; typical values are 0.58 m and 0.38 m. If your stand pivots the bicycle somewhere other than the bottom bracket (a cradle under the chainstays, say), measure from the pivot instead.

4. **Protractor.** Fit the protractor disc to the top of the head tube and the pointer to the stem as in Sec. 10, with the pointer on 0° when the wheel is straight. Mark P and G on the platform anyway (lay the 1 m rule along the head tube down to the platform for P; G is directly below the axle): the distance GP is a check on the trail, and the marks show how the contact point walks across the platform as the bars turn.

5. **Dial indicator.** Set the magnetic base on something heavy that does not move: a steel plate on a laboratory stool, or the base of a cast-iron stand, placed beside the head tube. Do not attach the base to the bicycle. Arrange the arm so that the plunger is vertical and its tip rests on the top of the stem cap. Where on the cap does not matter, since the cap is a plane perpendicular to the axis and turns in itself, but the cap is tilted 19° from horizontal, so use a flat or ball-ended contact point rather than a sharp one, which would skate. Compress the plunger about 15 mm so that it can follow the cap both down (the drop, a few millimeters) and up (when you lift the front end to turn the bars, step 1 of the procedure).

6. **Zero.** Zero the dial with the bars straight. Lift the front end a centimeter, set it down, and check that the dial returns to zero within 0.02 mm. If it does not, the base is moving or the tire is not settling; a smooth platform helps.

Procedure

1. For each steer angle, *lift, turn, set down, read*: lift the front end about a centimeter by the fork crown (the bicycle pivots on the stand), turn the bars to the next 10° mark on the protractor, lower the tire gently onto the platform, let go, and read the dial. Lifting means that the tire is placed fresh at each angle instead of being scrubbed sideways across the platform, which would wind up the sidewall and hold the frame a fraction of a millimeter off its geometric position.
2. Go left in 10° steps from 0° to the stop (usually 70°–80°), recording δ and the dial reading d on data sheet 2. Add readings at 5° and 15°; the small angles fix the slope.
3. Return to zero and confirm that the dial reads zero. Repeat to the right. Left and right should agree; their average cancels a small lean of the frame, and their difference is a check on how upright the bicycle was.
4. Take a second pair of runs. With care the readings repeat to 0.05 mm.
5. Watch the dial between 50° and the stop. The reading should reach a maximum and then come back before the stop, by only a tenth or two of a millimeter on the dial if the stop is at 75° and by nearly 0.4 mm if it is at 80°; that reversal is the flop angle δ_*, and it is worth bracketing with readings every 5° from 55° to the stop.
6. Note the angle-gauge reading at the largest δ. It should be a few tenths of a degree above the α you set; that is the frame pitching, $\varepsilon \simeq \Delta h / x_G$, and it is the same information as the dial reading, seen on a cruder instrument.

What to expect

Figure 5 shows the prediction for a road bicycle with $\alpha = 71°$, $R = 345\,\text{mm}$, $\rho = 11.5\,\text{mm}$, and $o = 40\,\text{mm}$, the geometry of the worked example of Sec. 5. The drop is 2.7 mm at 30° and 7.4 mm at the flop angle, $\delta_* = 69.5°$; the small-angle line overshoots by 11% at 30° and by 59% at 60°. At the stem cap, with $x_G/x_{\text{cap}} \simeq 1.5$, the dial sees two-thirds of these numbers, so the full run spans 4.4–4.9 mm on the dial, comfortably inside its 25 mm travel and over 400 divisions of 0.01 mm.

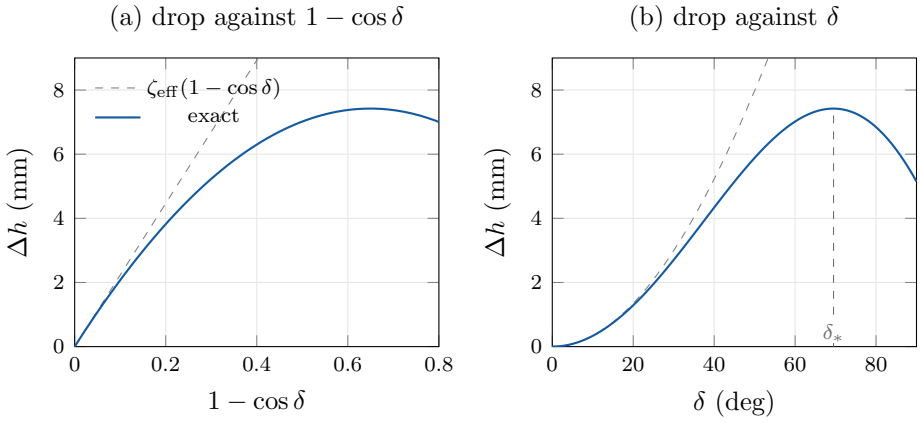


Figure 5: The prediction for the geometry of the worked example: exact law (solid) and small-angle rule (dashed). The exact curve peaks at the flop angle δ_* and comes back; the rule keeps climbing. Your points should follow the solid curve, a few percent low.

12 Analysis

Average the left and right readings at each $|\delta|$ (the average cancels a small lean of the frame, which adds an odd-in-δ term; the difference is a measure of the lean). Convert to Δh with Eq. (11). Then:

1. Plot Δh against $1 - \cos\delta$. Fit a straight line through the origin to the points with $\delta \le 20°$; its slope is ζ_{eff}, in the units of Δh. Three or four points at 5°, 10°, 15°, 20° fix it well; the drop at 20° is only about 1 mm, so the dial's 0.01 mm resolution is what makes this work.
2. Plot Eq. (6) with the measured R, ρ, o, and α on the same axes. It has no free parameters. The data should follow it, a few percent low we expect, and fall below the straight line past 20°.
3. Alternatively, fit Eq. (6) to all the points with ζ_{eff} as the single free parameter by letting o float (the script below does this). The fitted o should agree with the measured one to a few millimeters.
4. Read δ_* as the angle of the largest Δh, bracketed by the 5° readings between 55° and 75°, and compare with Eq. (8).

Expect the measured ζ_{eff} a few percent below the geometric value, since tire squash under load and fork flex both, we believe, reduce the drop.

MATLAB. The following function does steps 1–4; it is also supplied as `fit_drop.m`. Call it as `fit_drop(delta_deg, d_mm, xG, xcap, alpha_deg, R, rho, o)` with the geometry in meters.

```
function out = fit_drop(delta_deg, d_mm, xG, xcap, alpha_deg, R, rho, o)
% Drop experiment: lever correction, small-angle slope, exact-law fit.
% Notation as in "Bicycle wheel flop": alpha is the head angle from the floor.
ca   = cosd(alpha_deg);  sa = sind(alpha_deg);
del  = delta_deg(:)*pi/180;
dh   = d_mm(:)*xG/xcap;                        % drop at the tire, mm
x    = 1 - cos(del);
small = del <= 20*pi/180 & del > 0;
zeff_slope = (x(small)'*dh(small))/(x(small)'*x(small));   % through origin
zeff_geom  = 1e3*ca*((R-rho)*ca - o);
drop = @(oo,dd) 1e3*((R-rho)*(1-cos(asin(ca*sin(dd)))) ...
                     - oo*ca*(1-cos(dd)));                  % Eq. (22) with R -> R-rho
ofit = fminsearch(@(oo) sum((drop(oo,del) - dh).^2), o);  % let o float
zeff_fit = 1e3*ca*((R-rho)*ca - ofit);
beta  = o/((R-rho)*ca);
dstar = acos(beta*sa/sqrt(1 - beta^2*ca^2))*180/pi;       % Eq. (27)
[~,i] = max(dh);
out = struct('dh_mm',dh,'zeff_slope_mm',zeff_slope,'zeff_geom_mm',zeff_geom, ...
             'o_fit_mm',1e3*ofit,'zeff_fit_mm',zeff_fit, ...
             'dstar_pred_deg',dstar,'dstar_obs_deg',delta_deg(i));
dd = linspace(0,max(del),200);
plot(1-cos(dd), drop(o,dd), 'b-', 1-cos(dd), zeff_geom*(1-cos(dd)), 'k--', x, dh, 'ro');
xlabel('1 - cos \delta'); ylabel('\Delta h (mm)');
legend('exact, measured geometry','\zeta_{eff}(1-cos\delta)','data','Location','southeast');
end
```

13 Questions

1. You measured the head angle with an angle gauge and with a triangle built on the 1 m rule. Estimate the uncertainty of each from the resolution of the instruments and the lengths involved (how long a side does the triangle need for a 0.1° reading?), and say which you trust more and why. What systematic error does each have that the other does not?

2. You measured R from the floor to the axle and by rolling the wheel one turn. Which is larger, and why? Which is the right radius to put into Eq. (6), which was derived for a rigid wheel? How would the answer change if the rider were aboard?

3. The steer angle can be read on the head tube (as δ), on the floor with a pointer on the fork (as δ_p), or along the tire's footprint (as δ_g). For the example geometry, how far apart are the three at 45°? Which is easiest to read to 0.5°, and which is the one the theory wants? If you had only the floor protractor, how would you mount the pointer so that the conversion to δ is simplest?

4. The drop could also be measured with the dial indicator resting on the fork crown instead of the stem cap, or with a feeler gauge under the tire with the frame clamped so that it cannot pitch, or with a laser distance sensor aimed at the head tube. For each, what would you gain, what would you lose, and what would you have to correct for?

5. Why does averaging the left-turn and right-turn readings cancel a small lean of the frame? What does the difference between them measure?

6. We have said that tire squash and fork flex both make ζ_{eff} read low. Argue which way each one goes, and decide whether either could make it read high.

7. The frame pitches nose-down by $\varepsilon \simeq \Delta h / x_G$ during the run, which steepens the head angle by ε. Using "13% per degree," estimate how much this changes the drop at δ_*, and whether it is worth correcting for.

8. Suppose the platform had been left too low by 10 mm, so that the frame was nose-down by $10/580$ rad throughout. By what factor would the measured ζ_{eff} be wrong, and in which direction?

9. If the bicycle were instead held rigidly, unable to pitch, the tire would lift off the platform as the bars turn. By how much, and how would you measure it? Would the dial indicator on the stem cap read anything?

10. The drop is tiny, a few millimeters. Multiply it by the weight on the front wheel with a rider aboard (about 300 N) and compare the energy released with the work needed to turn the bars against a typical headset's friction. Which wins, and what does a rider feel?

Data sheet 1: geometry

Bicycle: _____ Tire: _____ Pressure: _____ Date: _____

Measured with the bicycle standing on both wheels on the floor.

Quantity	Reading 1	Reading 2	Average	Notes
Head angle α, gauge (deg)				zeroed on the floor; both sides
Head angle α, triangle (deg)				$h =$ b or $L =$
Front radius R, floor to axle (mm)				tire inflated
Front radius R, rolled (mm)				distance for one turn $/2\pi$
Tire width 2ρ (mm)				caliper at widest; footprint width:
Fork offset o (mm)				both sides
Trail GP on the floor (mm)				direct check

Adopted values: $\alpha =$ _____ deg $R =$ _____ mm $\rho =$ _____ mm $o =$ _____ mm (say which method, and why)

Computed: $\tau =$ _____ mm (agrees with GP?) $\tau_m =$ _____ mm o from the trail, $R\cos\alpha - \tau\sin\alpha =$ _____ mm

$\zeta = \cos\alpha\,(R\cos\alpha - o) =$ _____ mm $\zeta_{\text{eff}} = \cos\alpha\,[(R-\rho)\cos\alpha - o] =$ _____ mm $\beta =$ _____ $\delta_* =$ _____ deg

In the stand (Sec. 11, Setup):

Head angle set in the stand: _____ deg (must equal α above) shims under platform: _____ mm

Pivot of the stand (bottom bracket / other: _____) $x_G =$ _____ mm $x_{\text{cap}} =$ _____ mm lever factor $x_G/x_{\text{cap}} =$ _____

Steer angle read as: δ (head-tube disc) / δ_p (floor, pointer) / δ_g (floor, footprint) GP marked on the platform = _____ mm (agrees with τ?)

Data sheet 2: the drop

Dial zeroed at $\delta = 0$, bars straight. Lift-check at zero: returns to _____ mm. Return zero after each run: _____ _____ _____ _____

Procedure at each angle: lift the front end, turn to δ, set down, let go, read. Dial readings d in mm.

δ (deg)	$1 - \cos\delta$	L1	R1	L2	R2	mean d	$\Delta h = d\,x_G/x_{\text{cap}}$	predicted, rule, Eq. (7) Eq. (6)
0	0.000							
5	0.004							
10	0.015							
15	0.034							
20	0.060							
30	0.134							
40	0.234							
50	0.357							
55	0.426							
60	0.500							
65	0.577							
70	0.658							
75	0.741							
stop: _____								

Slope of Δh against $(1 - \cos\delta)$, points $\leq 20°$: ζ_{eff} = _____ mm Fit of Eq. (6), o floating: o = _____ mm, ζ_{eff} = _____ mm Geometric (sheet 1): _____ mm

Angle of largest Δh (observed δ_*): _____ deg predicted: _____ deg Left−right difference at 30° (lean check): _____ mm

Angle gauge at largest δ: _____ deg $-\ \alpha$ set = _____ deg (pitch ε) $\Delta h_{\max}/x_G$ in degrees: _____ (should agree)