A chaotic lock-in amplifier

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(Received 20 July 2007; accepted 21 December 2007)

We describe a method for lock-in amplification that uses a chaotic reference signal and a synchronized receiver. This technique is compared with conventional lock-in methods using a periodic reference signal. The apparatus can be constructed in an undergraduate electronics lab using a light emitting diode for the transmitter and a photodiode for the receiver, and allows students to explore a novel measurement technique based on chaotic dynamics.
[DOI: 10.1119/1.2835055]

I. INTRODUCTION

A lock-in amplifier acts as a high-gain narrow-band filter and can be a great aid in recovering a small response signal buried in noise.[1] The basic building blocks of a lock-in amplifier consist of amplifiers, filters, and phase-sensitive detectors and can be implemented with either analog or digital electronics.[2] A simple diagram for the signal path in a conventional lock-in amplifier is shown in Fig. 1. The reference signal of a conventional lock-in amplifier is a periodic signal with a narrow power spectrum.

In this paper we describe the construction of a lock-in amplifier that uses a chaotic reference signal. Because a chaotic lock-in amplifier uses a broadband reference signal, it might have some advantages in terms of signal capture time compared with a conventional lock-in amplifier, which typically makes use of a swept-sine method to capture the response signal over a wide-bandwidth.[3] We describe here how a lock-in amplifier can be built, we do not discuss possible applications.

The key ingredient of a lock-in amplifier is a phase-sensitive detector, so we address the related questions of how to "phase-lock" the stimulus and response signals from the chaotic lock-in amplifier, and how to modulate and demodulate the stimulus and response signals making use of the chaotic reference. The inspiration for this measurement technique comes from recent discoveries showing how to synchronize chaotic systems—the phenomenon known as chaotic synchronization.[4] There is now a vast amount of literature describing how to exploit chaotic synchronization for communication systems.[5] There appears to have been less work in applying recent discoveries in nonlinear dynamics to new solutions for problems in measurement and sensing.

The basic schematic for a simple chaotic lock-in amplifier is shown in Fig. 2. This design mimics the conventional lock-in amplifier by mixing a signal from the chaotic oscillator with the stimulus signal, looking at the sum frequency for up-conversion, and demodulating by multiplication and filtering for down-conversion.[6] This naive approach works in that it is possible to recover the amplitude of the response signal. However, it suffers from substantial nonlinear distortion which cannot be easily removed. The source of the in-band distortion is the sum and difference products generated by the broadband harmonics of the chaotic oscillator with the stimulus signal. Section II discusses a method to improve the recovered signal fidelity by creating an inverse system filter to remove the in-band signal contributions from the reference channel.

II. INVERSE SYSTEM DESIGN FOR A CHAOTIC LOCK-IN AMPLIFIER

To recover the response signal and to simultaneously systematically correct for nonlinear distortion, we use an inverse system approach for the design of the modulation and demodulation of the stimulus and response signals.[7] The basic schematic for this design is shown in Fig. 3.

The inverse system approach is a method for inverting the nontransient behavior of a nonlinear dynamical system and is a generalization of the method for inverting linear dynamical systems.[8] Inverse systems have been proposed for chaotic communications and the set-up usually requires a more complicated transmitter and receiver design than that required for the lock-in amplifier described here, because the modulating carrier for communications applications often needs to be regenerated by the receiver.[9] For lock-in amplifier amplification applications, we often have direct access to the reference signal modulating the stimulus signal. In a conventional lock-in amplifier this signal would be a periodic modulation signal generated by the lock-in amplifier itself. Alternatively, many lock-in amplifiers also provide an input port for the modulating signal, which is then used for demodulating the response signal. The demodulation process described here must undo the chaotic modulation in the response signal. The main advantage of the inverse system design is that the modulation/demodulation process is (conceptually) straightforward and can systematically correct for the complex in-band nonlinear distortion products produced by mixing the

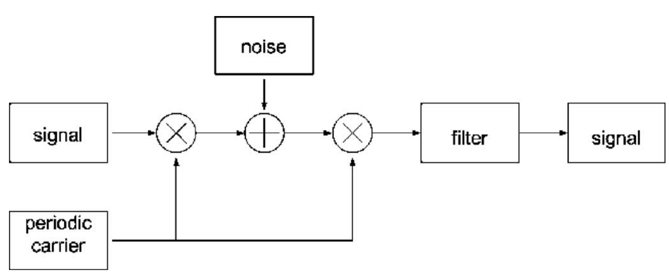

Fig. 1. Schematic for signals in a conventional lock-in amplifier.

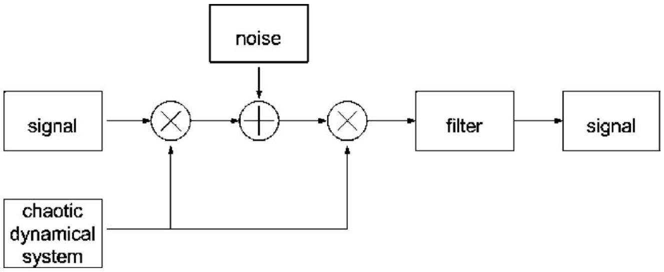

Fig. 2. Schematic for signals in a simple chaotic lock-in amplifier.

signal and reference. A disadvantage is the lack of reliability of the method with respect to noise (see Sec. IV).

Instead of transmitting a periodic reference signal for the chaotic lock-in, we transmit one or more of the time-domain signals of the state variables sampled from the reference signal, that is, from the chaotic dynamical system which is generating the reference signal. The reference signal for our chaotic lock-in differs in some significant ways from that for a conventional lock-in. Instead of direct multiplication of the stimulus and reference signal for up-conversion—conventional mixing—the mixed stimulus and reference signal is also used to generate the reference signal, a process we call differential mixing or differential up-conversion to distinguish it from multiplicative mixing (see Sec. III). That is, the stimulus signal is coupled into the reference signal generator. The transmitted signal sent to the channel or used to characterize the response of a sample in the channel is the mixed product of the stimulus signal and (stimulus-dependent) reference signal. For example, if the stimulus signal is generated by a light-emitting-diode (LED), then instead of periodically chopping the optical beam, we chaotically modulate the intensity of the optical beam by the reference signal by either using a variable polarizer or by directly modulating the current source of the LED.

Up to the differential mixing of the reference for modulating and demodulating, the proposed chaotic lock-in amplification method superficially resembles that of a conventional lock-in in that we rely on conventional mixing and filtering for signal recovery. Not shown in Fig. 3 are some additional features that are easily added. For instance, the chaotic reference signal is usually designed around a desired center frequency and then band-limited around this frequency using a tunable linear filter. Previous work has shown that this process does not adversely affect signal recovery and synchronization.[10] For quadrature modulation we can also add a Hilbert transform filter to the reference generator and use it to generate both quadrature components needed for the complex reference signal.[11] This feature is crucial for applying the chaotic lock-in to measure both the linear magnitude and phase response of the test device, sample, or channel.

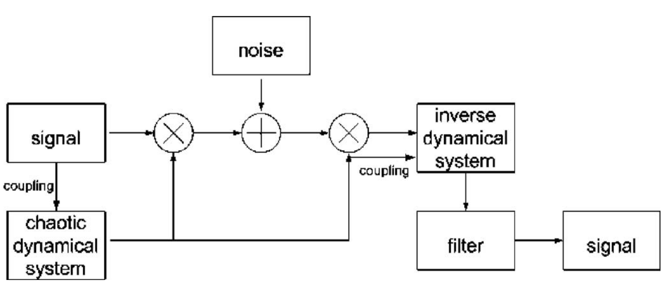

Fig. 3. Schematic for signals in a chaotic lock-in amplifier using an inverse system design.

III. CHAOTIC REFERENCE SIGNAL DESIGN

The chaotic reference signal is designed to include the following properties: it should transmit the input signal explicitly, which requires that the original system and the inverse system be the same order, that is, the same relative degree, preferably degree zero;[12] the transmitted and received signals should self-synchronize, which implies that the difference system design[13] should have a fixed point at the origin; the synchronization should be robust against small perturbations, which implies that the fixed point should be hyperbolic;[14] and the transients should decay quickly, which implies that the eigenvalues of the fixed point should be largely negative. We would also like a reference signal generator that can be implemented as a simple analog circuit. A survey of possible circuits from which to design these properties is given in Refs. 15 and 16.

To satisfy these conditions we designed a nonautonomous stimulus signal, $u(t)$, and a three-dimensional oscillator, $\mathbf{x}=(x_1,x_2,x_3)$, with the following form:[7]

Modulator equations:

$$\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} + \mathbf{b}\cdot f(\mathbf{x},u), \tag{1}$$

$$y = \mathbf{c}^T\cdot\mathbf{x} + f(\mathbf{x},u), \tag{2}$$

$$f = (x_1 + \mathrm{DC})^2\cdot u; \tag{3}$$

Inverse demodulator:

$$\dot{\mathbf{z}} = \mathbf{A}\cdot\mathbf{z} + \mathbf{b}\cdot(y - \mathbf{c}^T)\cdot\mathbf{z}, \tag{4}$$

$$\tilde{u} = f^{-1}(\mathbf{z}, y - \mathbf{c}^T\mathbf{z}); \tag{5}$$

Difference system:

$$\dot{\mathbf{d}} = (\mathbf{A} - \mathbf{b}\cdot\mathbf{c}^T)\cdot\mathbf{d}, \tag{6}$$

$$\mathbf{d} = \mathbf{x} - \mathbf{z}, \tag{7}$$

where $u(t)$ is the stimulus signal and $(x_1+\mathrm{C})$ is the modulation signal. The term C is a constant offset. The modulator x_1 is a chaotic oscillation which can be sent through a band-limiting filter so that it has a well-defined center frequency that is typically much greater than the center frequency of the stimulus signal $u(t)$. The bandwidth chosen for the filter is application dependent. For large signal capture it can be a low-pass filter with a cut-off determined by the Nyquist frequency of the receiver, as is used here with a cutoff of 50 KHz. Other applications could use a band-pass filter centered on a signal of interest. The performance of the system depends on the details of the filter. The impact of the filter on the lock-in characteristics will not be explored here, although simple experiments with filters of different bandwidths support this expectation.

The transmitted signal after up-conversion is $(x_1(t)+\mathrm{C})u(t)$. The response signal after down-conversion is $f=(x_1(t)+\mathrm{C})^2u(t)$. For our purposes it is sufficient to consider the simplest case, where $\mathbf{c}=\mathbf{0}$ so that $y=f$. The function f^{-1} is the inverse of f with respect to u. This response signal is then used as the forcing term to the inverse demodulator and this

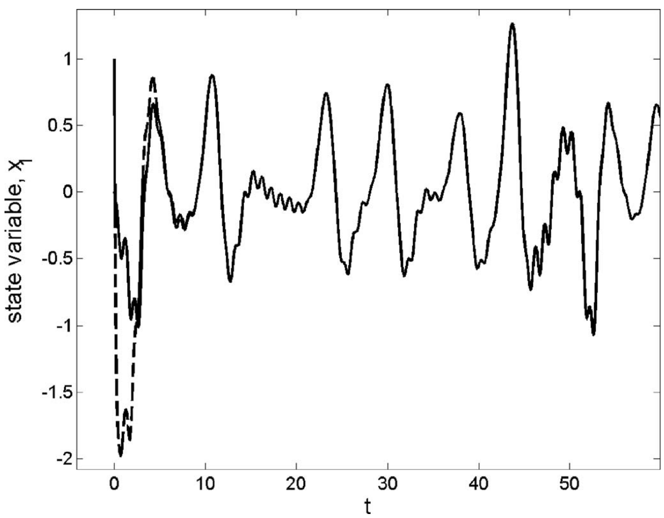

Fig. 4. Simulation showing the test signal up-converted with a chaotic reference signal and its synchronization with the receiver. The unsynchronized signal is represented by the dashed curve.

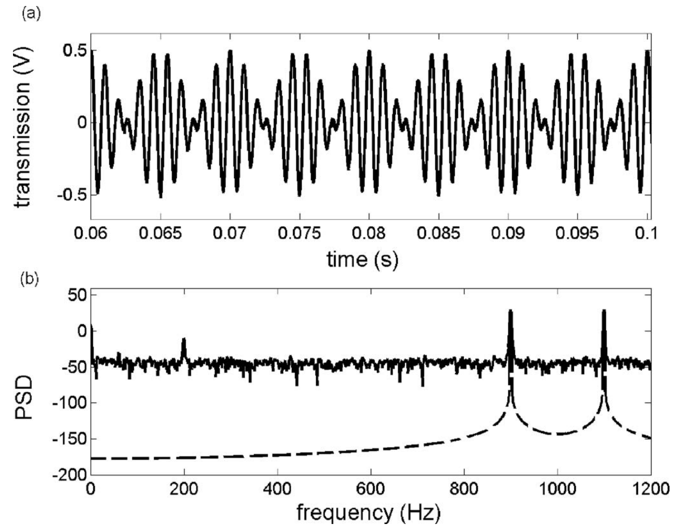

Fig. 6. (a) Modulated signal and (b) spectrum for periodic reference signal. The dashed curves in (a) and (b) and in Figs. 7–11 represent simulations.

system, when integrated and synchronized, produces $\tilde{u}(t)$, which should be the original stimulus signal $u(t)$ subject only to channel distortion and relatively uncontaminated by channel noise and mixer distortion.

As a first test of the feasibility of the method we did simulations where the channel is simply modeled by additive noise to the transmitted signal. Figure 4 shows the synchronization of the chaotically modulated signal with the receiver. Figure 5 shows a comparison of the test signal u and the output of the inverse system $\tilde{u}$. As hoped, the simulations show that synchronization is found after only a few oscillations and the inverse method at least qualitatively corrects for the distortion and results in a good match between the input signal and the demodulated signal.

IV. CHAOTIC LOCK-IN TEST APPARATUS

To test the feasibility of using a chaotic reference signal and an inverse system design for lock-in signal detection we built a simple electronic system for evaluating the lock-in measurement technique using a chaotic reference signal. Because we want a system with which we can make a direct comparison to conventional lock-in detection using a periodic reference signal, we built a "software lock-in," where the transmitted signal is created in software and sent through an electronic arbitrary waveform generator (essentially a digital to analog converter), to a light emitting diode, and then detected with a photo-detector and amplifier circuit, which is then converted back to a digital form with an analog-to-digital converter (A/D). Noise is added to the system by the transmission channel. The amount of noise added by the channel is determined by the distance between the transmitter and receiver and the amount of background lighting. Adding or removing noise is as simple as turning the room lights on or off. All the essential signal processing is done in software for both the periodic and chaotic reference signals, so that an almost direct comparison between different reference signal designs can be made.

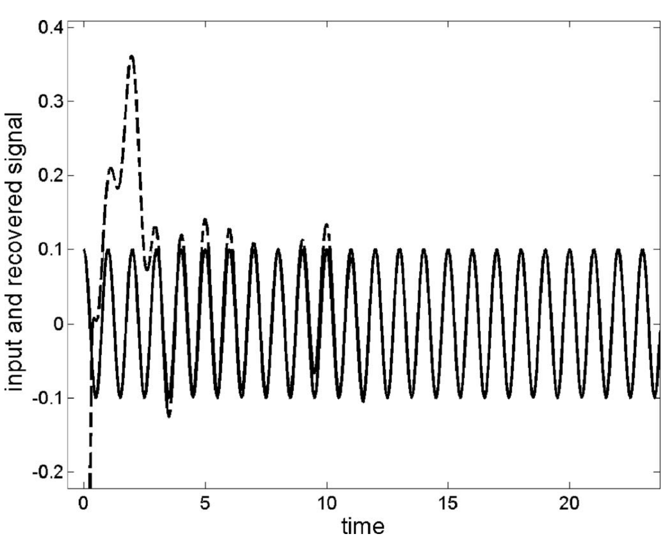

Fig. 5. Comparisons of the input test signal and demodulated test signal before synchronization (dashed curve).

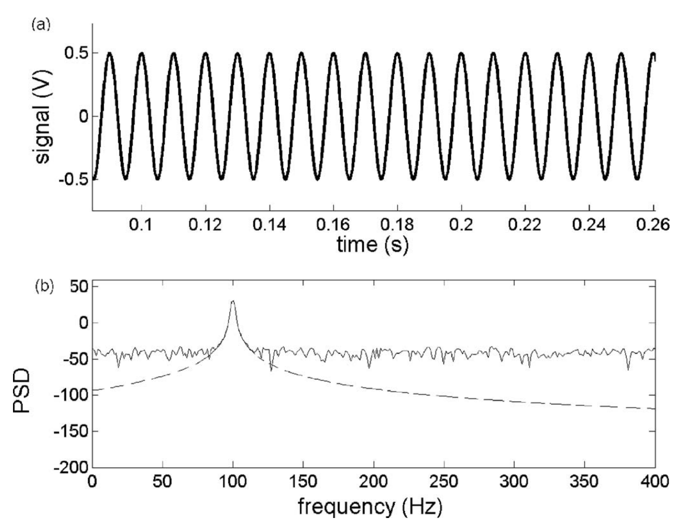

Fig. 7. (a) Recovered signal and (b) spectrum for periodic reference signal. The PSD of the recovered signal is about 50 dB above the simulation noise floor.

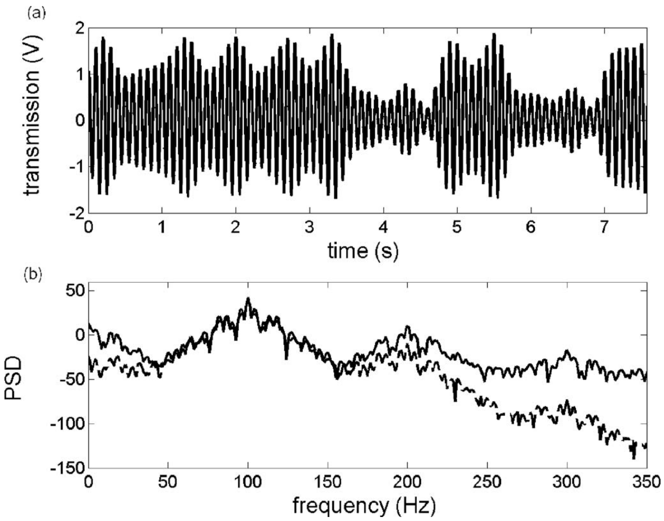

Fig. 8. (a) Modulated signal and (b) spectrum for chaotic reference signal. This shows the actual transmitted signal from which the information signal (in the cases studied here a single tone) is recovered as shown in Fig. 9.

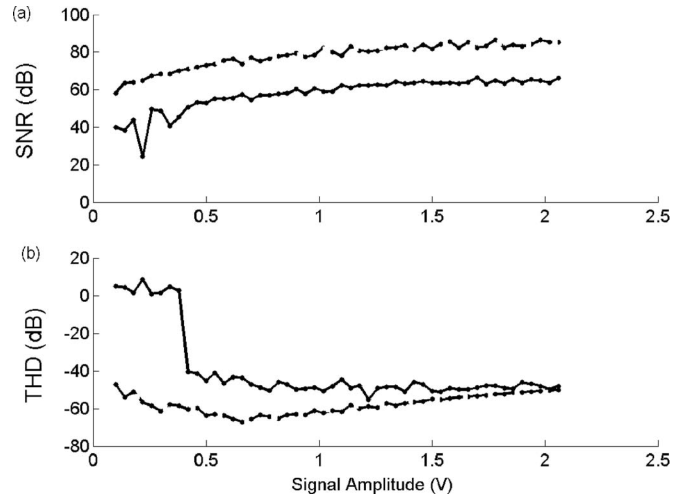

Fig. 10. Standard lock-in figures of merit. (a) Comparison of the recovered signal-to-noise ratio for a simulation (dashed line) and experimental (solid line) periodic lock-in reference signal. (b) Comparison of the THD, a measure of the nonlinear distortion in the transmitted signal—the power in all the harmonics outside the (first harmonic) transmitted signal.

The optical transmitter is a red LED with appropriate electronics for variable biasing. The optical receiver is a photodiode and amplifier which is sensitive to the visible and infrared spectrum (600–1000 nm). The control and data collection is done with A/D and digital-to-analog boards with 16 bit dynamic range and data collection is in the range of 50 KHz. Calibration methods are used to compensate for any nonlinearity in the transmitter and detector, and noise is easily introduced into the channel by changing the distance between the transmitter and receiver. A direct comparison is made between difference lock-in schemes by keeping the system configuration identical when comparing different lock-in techniques. Additional details about the experimental testbed are in Ref. 17.

To compare the two different lock-in methods, we estimated the signal-to-noise (SNR) ratio and total harmonic distortion for each received signal. For the lock-in using a periodic reference signal, the transmitted signal is a sine wave with a frequency of 100 Hz and the reference signal is a sine wave with a frequency of 1000 Hz. The modulated and recovered signals are shown in Figs. 6 and 7. The recovered signal and spectrum show that the signal-to-noise ratio is about 70 dB above the noise floor with little harmonic distortion. The analogous experiment for a chaotic lock-in is shown in Figs. 8 and 9. An examination of the power spectral density (PSD) of the recovered signal shows that there is a bit more harmonic distortion and the signal is about 55 dB above the noise floor.

Note that the chaotic lock-in method with distortion correction using an inverse system design is able to both synchronize and accurately recover a clean input signal in the presence of significant channel noise. The fact that a typical signal-to-noise ratio for a broadband method is not as high as

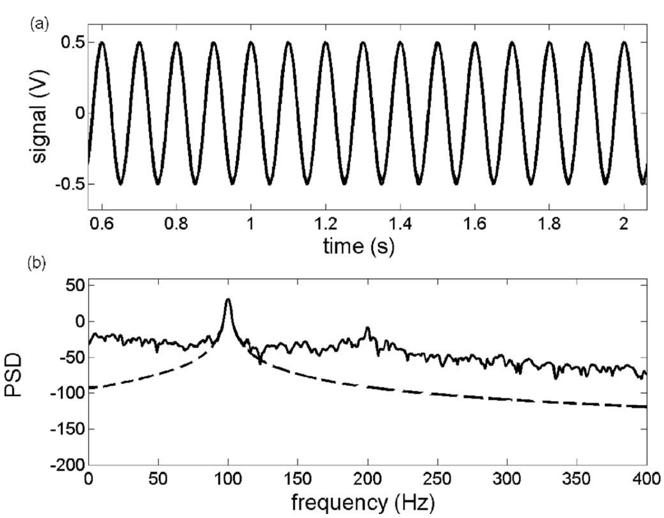

Fig. 9. (a) Recovered signal and (b) spectrum for chaotic reference signal. The difference is not detectable in the time domain. Impairments in the recovered signal (solid line) are easier to detect in the frequency domain.

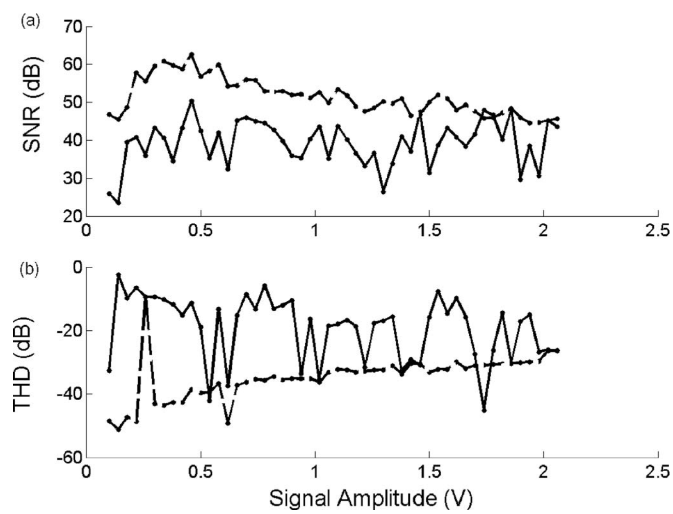

Fig. 11. Chaotic lock-in figures of merit. (a) Comparison of the recovered signal-to-noise ratio for a simulation (dashed line) and experimental (solid line) chaotic lock-in reference signal. (b) Comparison of the (THD, the abrupt swings in the experimental (solid curve) results are due to a loss of synchronization in the receiver.

a narrowband method is not surprising. In general, we expect a trade-off in the signal-to-noise performance for broadband coverage.

Figures 10 and 11 show the measured SNR and the total-harmonic-distortion (THD) of each method for increasing signal level. The SNR [Fig. 11(a)] of the chaotic lock-in method is generally worse than the single frequency method. However, it can still be useful in situations requiring the rapid capture of a broadband of frequencies. Of greater concern is the overall quality of the chaotic lock-in performance which appears erratic in comparison to the single frequency method [Fig. 11(b)]. A more detailed examination of this erratic performance reveals that it is associated with intermittent loss of synchronization. The conclusion from the graphs is that, when the chaotic lock-in system achieves and maintains synchronization, it provides acceptable lock-in performance, but the current design does not adequately provide for reliable synchronization. We are exploring modifications of our design in order to achieve a more stable and robust synchronization in the presence of significant channel noise.

ACKNOWLEDGMENTS

We wish to acknowledge the support of the Agilent Labs Grass-roots Research Program, which funded the first author's work on this project as part of an internship at Agilent Labs. We also thank Greg VanWiggeren for his assistance with the early part of this project.

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[17] See EPAPS Document No. E-AJPIAS-76-017803 for additional details of the measurement apparatus. This document can be reached through a direct link in the online article's HTML reference section or via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html).