

decency, brotherhood, and peace? All of these men can be admired for their scientific accomplishments, but for probity and honor I believe Isaac Newton stands over them as if still on the “shoulders of giants.”

The conflict with Flamsteed is not simply the vengeful act of a self-centered Newton trying to cheat a poor astronomer out of his data. Newton and Flamsteed differed sharply in their perceptions of Flamsteed’s official position—and it seems to me, by the standards of today at least, that Newton’s viewpoint is the more sustainable. To Newton, the Astronomer Royal was a civil servant, a scientist paid by the State to obtain results of use to the State. Is it not, then, almost a malfeasance of duty for a government scientist to refuse to relinquish data until such time as he can complete some personally conceived magnum opus? Think of this conflict in contemporary terms: How sympathetic would we be to a scientist working for NIST, or the National Oceanic and Atmospheric Administration (NOAA), or the National Aeronautics and Space Administration (NASA), or any of a number of other government agencies with responsibility to provide the public with critically needed information, if that scientist were to refuse to release that information until such time far in the future as suited his own personal agenda? Newton was no saint, and he certainly had his personal reasons for singlemindedly pursuing Flamsteed’s lunar data. Nevertheless, the manner of this pursuit—via the communication channels available to him as an influential scientist and administrator—does not, in my view, justify the claim in the subtitle of the book that Newton suppressed his antagonists’ scientific discoveries. Quite the contrary in the case of Flamsteed, he was trying—even if for his own somewhat venal reasons—to “liberate” those observations and make the work available for public use. In the case of Gray’s discoveries, Newton may not have facilitated their publication, but he certainly did not—and could not—prevent Gray from communicating his observations in letters to other natural philosophers, which was the usual manner of the day.

But don’t take my word for this. Read the book yourself and let Clark and Clark take you back to a uniquely colorful place and time of extraordinarily talented scientists with all-too-human frailties.

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The Topology of Chaos. Robert Gilmore and Marc Lefranc. 596 pp. Wiley, New York, 2002. Price: \$84.95 ISBN 0-471-40816-6. (Nicholas B. Tufillaro, Reviewer.)

One Ring to rule them all,
one Ring to find them,
one Ring to bring them all
and in the darkness bind them
Lord of the Rings, J.R.R. Tolkien

A *knot* is an embedding of a circle, a ring, into three-dimensional space. There are an infinite number of ways in which this can be done. The figure eight knot, the trefoil knot of celtic lore (a stylized version of the shamrock), and of course the “unknot,” a simple closed loop—these are just three of the more common examples of an embedded ring. When two or more knots are entangled together they form a *link*. Knots and links enter dynamics through the search for periodic solutions to the equations of motion. A periodic orbit that arises by solving a system of three ordinary differential equations is a knot; a collection of such orbits forms a link. For instance, integrable Hamiltonian systems with two degrees of freedom in configuration space can be described by a flow in a three-dimensional phase space, and the periodic orbits of this flow form a braid of “torus” knots. This concise topological description of a dynamical phenomenon follows from the fact that solutions of such integrable systems, in appropriate action-angle variables, can be viewed as rational (periodic) or irrational (quasi-periodic) windings on a torus. In this instance topology and dynamics are really the same coin viewed from different sides. Although beautiful in mathematics and celtic art, torus knots and their links are considered tame by dynamists. More exciting links are generated by looking for periodic orbits embedded in strange attractors found in three-dimensional dynamical systems like the Lorenz, Duffing, or Rössler equations.

Recall the familiar picture of the “Lorenz attractor,” so often seen on computer screens, consisting of two spiraling branches. A single chaotic trajectory of the Lorenz attractor comes arbitrarily close to an infinite number of unstable periodic orbits. Informally, a chaotic attractor can be defined as the closure of all the periodic orbits, or knots, it contains. In order to get a handle on these “Lorenz knots,” the mathematicians Birman and Williams introduced a topological caricature, a cartoon, for the Lorenz attractor, called a *template*—which is simply a collection of two-dimensional surfaces called branches, which are bent, twisted, and glued together in such a way so as to resemble the two spiraling wings that are generated by the Lorenz equations. By projecting out the stable directions of the flow in the Lorenz equations, Birman and Williams were able to show that the resulting flow induced on their template, or “knot-holder,” was a faithful topological representation for the knots that occur in the familiar Lorenz attractor. A number of physicists, led by Gilmore, realized that this process could be worked in reverse: periodic orbits could be extracted from experimental data and used to construct topological models—templates.

Moreover, these topological models carry important dynamical implications. For instance, the existence of a single so-called “postive entropy” orbit, extracted from a chaotic time series, provides strong evidence that an attractor is chaotic, in a way that is much easier to obtain than methods based on calculating metric properties of an attractor such as its fractal dimension. The simplest flows (as opposed to maps) that can exhibit chaotic behavior are three dimensional, and the simplest chaotic attractors are those with just one unstable direction (one postive Lyapunov exponent). It is exactly this class of attractors which can be modeled by templates. As Gilmore and Lefranc illustrate repeatedly, with data sets ranging from mechanical systems to lasers, what matters in successfully constructing such a model from experimental data is not the underlying number of dimensions of the system, but rather that the attractors generated can be faithfully embedded in three dimensions. Thus, what we

have is a physical theory and experimental procedures for identifying and classifying the “simplest” chaotic forms that can exist. The bulk of this book is a detailed user’s manual for how to extract these topological models from experimental data, and why such an analysis is both useful and important.

The book focuses on teaching the topological modeling methods developed during the past decade by Gilmore and his co-workers. A good grounding in an elementary text, such as Steven Strogatz’s *Nonlinear Dynamics and Chaos* (Addison–Wesley, 1994) should be enough to get the motivated reader started. The opening chapter of the book recounts the authors’ first encounters with the challenges posed in understanding nonlinear behavior in laser systems. This sets the stage for the questions they want to tackle: how to classify the geometric and topological mechanisms that generate chaotic behavior, and how to identify these forms in experiments. This first chapter contains a lot of the standard jargon of nonlinear dynamics (and some lesser known terms and concepts like “snakes”), such as “crisis,” “saddle-node bifurcation,” “Newhouse series,” and “Lyapunov exponents,” which could be off-putting to some readers. However many of these terms are described more fully in the following chapters when they are actually needed.

The beginning chapters are a nice summary of dynamical systems theory, with a strong bent toward topics and terms that are useful in the topological analysis to come. As one might expect, the Logistic map makes an early appearance, with a detailed examination of its symbolic dynamics via kneading theory. The emphasis is on understanding and calculations, as opposed to rigorous proofs, which can be found, for example, in Robert Devaney’s book [*An Introduction to Chaotic Dynamical Systems* (Addison–Wesley, 1992)]. With the analysis of “horseshoe maps,” the text begins to distinguish itself from “standard” treatments. The authors provide a description of the horseshoe map, as well as a “reverse” horseshoe, and show how it is topologically distinct from the Smale horseshoe studied in almost all books on chaotic dynamics. The text is peppered throughout with very insightful comments of the type that are often heard in a lectures, but never seem to make it into texts. For instance, when speaking of the symbolic reduction of the dynamics of the Smale horseshoe, the authors write, “To conclude, let us emphasize that the two-sidedness of symbolic sequences for the horseshoe map is not due directly to the dynamics being two-dimensional. Rather, it originates in the distinction between stable and unstable spaces, regardless of their dimension.”

After examining fairly standard material about maps, the authors look in detail at some typical flows: the Duffing, van der Pol, Lorenz, and Rössler systems. Each example is nicely introduced with a concrete physical realization. Again, in addition to standard material such as the analysis and stability of fixed points and periodic orbits, the authors also introduce some original ideas, such as the first steps toward a “structure” theory of dynamical systems. Most striking is the authors’ repeated emphasis on the role singularity theory can play in the classification of chaotic forms. This idea is implicit in Milnor and Thurston’s original presentation of kneading theory [“On iterated maps of the interval,” in *Dynamical Systems*, edited by J. Alexander, Lecture Notes in Math. Vol. 1342 (Springer-Verlag, Berlin, 1988), pp. 465–563], but the suggestions and initial steps toward pushing this idea beyond one or two dimensions are very intriguing.

The dynamics and topological structure of even the simplest two-dimensional maps are still a subject of active mathematical investigation. Recent studies, for example, on forcing relations in horseshoe maps, by Toby Hall and Andre De Carvalho (“Symbolic dynamics and topological models in dimensions 1 and 2,” to appear in *Topics in Dynamics and Ergodic Theory*), nicely complement some of the more physically motivated approaches described in this text.

The middle chapters present the core material about knots, braids, links, relative rotation rates, and templates, needed to set the stage for topological modeling, and are largely self-contained. Material that can be found scattered throughout several research papers is presented in a clear and coherent fashion. As with most of the text, there are ample figures, and care is taken to describe the meaning and motivation of the equations introduced. Although much of the mathematics will be new to many readers, the essential calculations can be mastered with a basic command of elementary knot theory (which is accessible to undergraduates), combinatorics, and algebra. Invariant manifold theory plays an important role in motivating the development of templates through the Brimant–Williams Theorem, but it is not essential for calculating knot invariants used to identify templates.

The next chapters show that this is not just a book about theory. The authors describe in great detail the signal processing tricks needed to get a good embedding and a topological model, and they go on to illustrate the results of these procedures on a diverse collection of experiments, ranging from the Belousov–Zhabotinskii chemical reaction to several different types of laser systems. Along the way they present ample evidence that certain topological forms such as the Smale horseshoe or the Lorenz attractor pop up again and again in a wide spectrum of physical systems. The remaining chapters in the middle part of the book draw much from the authors’ own original research, and begin to develop a theory for why these chaotic forms are so ubiquitous.

The book concludes with two chapters that are visionary. Drawing on analogies from the theory of Lie Groups and Singularity Theory, the authors outline how to extend the program for topological analysis beyond three dimensions. More importantly, they advocate a “program for dynamical systems theory,” with enough questions and insights to keep physicists and mathematicians busy for quite some time. These chapters alone may be worth the price of admission for the cognoscente.

The primary audiences for this text will be graduate classes and researchers in chaotic nonlinear dynamics. However, a motivated teacher can find material in the book of use to undergraduates. For instance, as part of an undergraduate research project I had one student develop a Mathematica notebook for computing relative rotation rates and plots of templates and their knots. Indeed, even some of the simpler topological models described in this text have yet to be completely explored or characterized, and might present some interesting challenges in undergraduate research.

It has been fourteen years from the time Robert Gilmore first began studying chaos in lasers to the publication of this book, which might be seen as book three of a trilogy. In book one, *Lie Groups, Lie Algebras, and Some of Their Applications* (Wiley, New York, 1974) Gilmore first explored the continuous group as a sort of generalized kinematics—the stage if you will, for all dynamics. In book two, *Catastrophe Theory for Scientists and Engineers* (Wiley, New York, 1981), he elucidated the role singularity theory can play in

understanding and classifying the equilibria of gradient dynamical systems—arguably the simplest dynamical systems with interesting properties. In these first two books he expertly demonstrated how a mature mathematical theory can present us with a more unified view of seemingly diverse dynamical phenomena. In this third book Gilmore and Lefranc step one more rung up the ladder of dynamical complexity to explore the simplest chaotic dynamical systems, and discover that to date there is only a partial mathematical theory to guide us. Nevertheless they urge us to explore further, leading us as far down the path as they can. This is a story that is still unfolding, and although the route may not be clear, there will no doubt be many more adventures along the way.

Not all who wander are lost
J.R.R. Tolkien

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Tritium on Ice: The Dangerous New Alliance of Nuclear Weapons and Nuclear Power. Kenneth D. Bergeron. 234 pp. MIT Press, Cambridge, MA, 2002. Price: \$25.00 ISBN 0-262-02527-2. (David Hafemeister, Reviewer.)

Tritium is a key component in the primary stage of a nuclear weapon. Without tritium–deuterium fusion boosting in the primary, the secondary will not explode with significant yield. Tritium does not add much energy to the primary’s fission yield, but it shortens the time. It takes about 80 doubling generations for a primary to explode, but by getting a quick dose of neutrons, generations can be skipped, saving time before violent disassembly. By increasing the fission efficiency of the ^{239}Pu or ^{235}U , less material is needed, and the size of primaries can be reduced.

The US has not produced tritium since 1988, relying instead on existing supplies. But these are being depleted by the spontaneous decay of tritium (half-life 12.3 yr), and the Department of Energy decided in 1998 to resume production, at the Tennessee Valley Authority’s (TVA) Watts Bar reactor. Kenneth Bergeron’s *Tritium on Ice* raises three main concerns about this decision: (1) The urgency of the need for tritium. (2) The breach in the traditional separation between military and commercial fuel cycles. (3) Reactor and environmental safety issues.

The need for more tritium: Bergeron correctly points out that the need for tritium is driven by US plans to maintain a large enduring stockpile of nuclear weapons. One might think that the 2002 Strategic Offense Reduction Treaty (SORT) would lead to a reduction to 2200 warheads, but the US desire for flexibility, with a large “responsive force” and a considerable number of spares, actually suggests a total of about 10 000 warheads. J. Cirincioni [Phys. and Soc. **31**, 14 (July 2002)] considers the following stockpile for the year 2012:

Operational deployed force	2200
Warheads on 2 Tridents in overhaul	240
Missile and bomber warheads in response force	1350
Nonstrategic bombs assigned to US/NATO	800
Nonstrategic cruise missiles (SLCMs)	320
Non-strategic spares	160
Intact warheads in inactive reserve	4900
Total	10 000

After considering proposals to make tritium in dedicated accelerators or reactors, the DOE opted to contract for tritium services at an existing TVA nuclear power plant. One can estimate tritium demand from three factors: the decay rate of tritium (mean life $\tau=17.7$ yr), the DOE’s proposed tritium production rates, and the number of warheads in a future stockpile. Tritium is produced from the absorption of neutrons by ^6Li in thermal reactors. DOE stated in 1998 that it would start tritium production of 2.5 kg/yr in 2005 for nuclear weapons under START I, which sustains some 8000 strategic warheads, plus other warheads. DOE said it would postpone production of 1.5 kg/yr until 2011 if START II entered into force with a limit of 3500, plus other warheads. Deeper cuts in warheads would relax tritium requirements, and postpone even further the need for new tritium. A factor of 4 reduction, from 10 000 to 2200, would extend the time before new supplies are needed by two half-lives, or a total of 25 yr. My estimate ignores the details of the actual tritium cycle (reserves, pipeline, recycle losses).

In 2005, under START I, the tritium needed in the stockpile under steady-state conditions would be

$$m_1 = \tau(dm/dt) = (17.7 \text{ yr})(2.5 \text{ kg/yr}) = 44 \text{ kg},$$

with an average tritium budgeted per warhead of about

$$44 \text{ kg}/10\,000 = 5 \text{ g}.$$

A reduction in warheads postpones the need for tritium production as follows:

SORT (START III) at 2000 warheads + 3000 reserves, tritium production begins in 2015.

$$m_2 = (5000 \text{ warheads})(5 \text{ g/warhead}) = 25 \text{ kg},$$

$$\begin{aligned} \Delta t &= -[\ln(m_2/m_1)](\tau) \\ &= -[\ln(25 \text{ kg}/44 \text{ kg})](17.7 \text{ yr}) \\ &= 10 \text{ yr} + 2005 = 2015. \end{aligned}$$

SORT at 2000 warheads: $m_3 = 10 \text{ kg}$, $\Delta t = 26 \text{ yr} + 2005$, begins in 2031.

SORT II at 1000 warheads: $m_4 = 5 \text{ kg}$, $\Delta t = 38 \text{ yr} + 2005$, begins in 2043.

Thus the urgency of the need depends critically on the number of warheads anticipated.

Separation of the military and commercial fuel cycles: The multilateral Nonproliferation Treaty is our only hope to prevent the global spread of nuclear weapons. The US, as one of the five nuclear weapons states (NWSs), has long promoted the ideal of separation between military and commercial fuel cycles. This is not a legal requirement, but rather a matter of setting an example to the 180 non-NWSs. The US mostly adheres to this constraint, but there have been exceptions, such as Hanford’s N reactor (shut down in 1988), which made military plutonium as it sold electricity. It is true that having the quasi-private TVA reactors make tritium would