RESOURCE LETTER

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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course content in specified fields. [The letter E after an item indicates elementary level or material of general interest to persons becoming informed in the field. The letter I, for intermediate level, indicates material of somewhat more specialized nature; and the letter A, indicates rather specialized or advanced material.] No Resource letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455.

Resource Letter: ND-1: Nonlinear Dynamics

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(Received 10 July 1996; accepted 13 March 1997)

This Resource Letter provides an introductory guide to the literature on nonlinear dynamics. Journal articles and books are cited for the following topics: general aspects of nonlinear dynamics and applications of nonlinear dynamics to various fields of physics, other sciences, and a few areas outside the sciences. Software and Internet resources are given also.

I. INTRODUCTION

Nonlinear dynamics and its subdiscipline ‘‘chaos theory’’ have swept over the landscape of science, mathematics, and engineering in the past two decades. The subject has benefited from some good public relations (Ref. 75), but its staying power is rooted in the universality of its methods and results: Although each nonlinear system may be quite distinct in its detailed behavior, most nonlinear systems fall into broad classes with common qualitative and quantitative features. Many of the theoretical and experimental advances in nonlinear science, moreover, can be explored with only a rudimentary background in mathematics and a pocket calculator or with simple computer programs (Refs. 42, 67, and Sec. VII). Experiments, such as the forced vibrations of a simple pendulum (Ref. 12), a bouncing ball (Ref. 27), or a semiconductor diode circuit (Sec. VI B), are straightforward and inexpensive. Because nonlinear effects are observed in everyday phenomena—such as a dripping faucet (Sec. VI I) or avalanches in sandpiles (Sec. VI F)—they capture the imagination of students at all levels.

Nonlinear dynamics can appear at several points in the undergraduate or graduate curriculum. It can be included as a special topic in a standard course in classical mechanics (Ref. 79) or differential equations (Ref. 48). Courses focusing on nonlinear dynamics at the junior, senior, or beginning graduate level are also becoming common (Sec. IV B). Several books aim to introduce nonlinear dynamics to students who are not necessarily science majors (Sec. IV A and Ref. 10).

There are two important themes in nonlinear dynamics. One emphasizes the temporal behavior of systems, most with no significant spatial variation. The other emphasizes spatial structures and the formation of spatial patterns. Most beginning courses in nonlinear dynamics focus on the temporal behavior of ‘‘low-dimensional’’ systems, that is, those governed by a few degrees of freedom, for which there now exists a fairly complete body of theoretical results and experimental techniques (Sec. IV B and Ref. 90). Methods of nonlinear time-series analysis (Sec. V D and Ref. 53), in which a sequence of values of a single dynamical variable is used to determine the qualitative and quantitative measures of the temporal dynamical behavior of the entire system, have dominated both the theoretical and experimental methodologies and are already finding their way into many practical applications.

The extension of these notions to higher-dimensional systems, to problems of ‘‘spatiotemporal’’ chaos, and to the question of turbulence is still a subject of active debate and inquiry (Sec. VI F). So too is the question of the correspondence between chaos in classical and quantum-mechanical systems (Sec. VI J). Both issues, though, have motivated some exciting new experimental and theoretical work.

The crucial theoretical construct in nonlinear dynamics is phase space (also called state space). Each of the (independent) dynamical variables is used as a coordinate to construct the state space for the system. For a deterministic system, the future behavior is determined by the current state of the system, represented as a point in state space. As a system evolves in time, its state-space representation maps out a trajectory in that space. Sets of trajectories form a phase portrait. These phase portraits often have interesting geometric properties. For example, chaotic systems can have phase portraits with a fractal geometric structure.

The important and distinctive features of nonlinear behavior are as follows:

- Symmetry-breaking, either temporal or spatial. The temporal response of a system need not be the same as that of the ''driving force.'' Even autonomous systems (those with no explicit time-dependent forcing) can spontaneously develop complex temporal behavior. The most dramatic of these broken symmetries is chaotic behavior, which is bounded, completely aperiodic behavior. For nonlinear systems with significant spatial variation, the spatial patterns may be independent of the boundary conditions.
- Dramatic changes in behavior, called bifurcations, which occur over extremely small parameter ranges. There are several common classes of bifurcation events—including period doubling, intermittency, and crises—that organize the transitions between regular (periodic) behavior and chaotic behavior.
- Sensitive dependence on initial conditions (the so-called Butterfly Effect) (Refs. 5 and 9): Small changes in initial conditions may lead to qualitatively and quantitatively different long-term behavior. Such sensitivity leads to the loss of long-term predictability even if the system is completely deterministic.
- Universal scaling laws for the transitions between chaotic and regular behavior.

Much of the analysis of the temporal behavior of nonlinear systems is carried out using time-series data from a single dynamical variable. The series may be generated by using a ''stroboscopic'' technique: Every time the state-space trajectory intersects a plane in the state space, forming a so-called Poincaré section, a data point is recorded. From such a series, the topology and dynamics of the entire attractor can be reconstructed. See the references in Sec. V C and the texts in Sec. IV B.

We have selected references that emphasize the breadth and connectedness of nonlinear dynamics at a level appropriate for someone new to the field, although we have included advanced-level citations for historically important papers or those that have played a significant role in the modern development of nonlinear dynamics.

II. JOURNALS

Journals with predominantly nonlinear dynamics articles:
Chaos
Chaos, Solitons and Fractals
Dynamics and Stability of Systems
Ergodic Theory and Dynamical Systems
Fractals
International Journal of Bifurcations and Chaos
Journal of Nonlinear Science
Nonlinear Dynamics
Nonlinearity
Physical Review E (prior to 1993, Physical Review A)
Physica D

Rapid publication journals with a significant number of papers devoted to nonlinear dynamics:
Europhysics Letters
Physics Letters A
Physical Review Letters

Other journals that often have articles on nonlinear dynamics:
American Journal of Physics
Computers in Physics
European Journal of Physics
International Journal of Nonlinear Mechanics
Journal of Statistical Physics

III. CONFERENCE PROCEEDINGS

Several conferences on nonlinear dynamics are held regularly.

1. SIAM Conference on Applications of Dynamical Systems (every other year).

2. **Proceedings of the First Experimental Chaos Conference**, edited by S. Vohra *et al.* (World Scientific, Singapore, 1992).

3. **Proceedings of the Second Experimental Chaos Conference**, edited by W. Ditto *et al.* (World Scientific, Singapore, 1995).

4. **Measures of Spatio-Temporal Dynamics**, edited by A. M. Albano, P. E. Rapp, N. B. Abraham, and A. Passamante (North-Holland, Amsterdam, 1996).

IV. TEXTBOOKS AND EXPOSITIONS

A. Popularizations

5. **Chaos, Making a New Science**, J. Gleick (Viking, New York, 1987). A best-seller that chronicles the development of the scientific study of chaos. Includes good biographical sketches of many of the major players in the development of chaos. (E)

6. **Does God Play Dice? The Mathematics of Chaos**, I. Stewart (Blackwell, New York, 1989). For the scientifically literate reader. Emphasizes the mathematical approach to chaos, but does pay some attention to experiments. (E)

7. **Order Out of Chaos**, I. Prigogine and I. Stengers (Bantam, New York, 1984). A wide-ranging look at the new paradigm: how order can develop from randomness in nonequilibrium systems. (E)

8. **Newton's Clock: Chaos in the Solar System**, I. Peterson (Freeman, New York, 1993). An excellent introduction to the history of nonlinear dynamics. (E)

9. **The Essence of Chaos**, E. N. Lorenz (University of Washington Press, Seattle, 1993). The text of popular lectures given by Lorenz, one of the pioneers of chaos. (E)

B. Introductory texts for science students

The following are listed more or less in order of increasing demands on the reader's mathematical sophistication.

10. **Chaos Under Control: The Art and Science of Complexity**, D. Peak and M. Frame (Freeman, New York, 1994). Intended for a course for first-year college students. (E)

11. **Understanding Nonlinear Dynamics**, D. Kaplan and L. Glass (Springer-Verlag, New York, 1995). Biological and medical orientation. (E)

12. **Chaotic Dynamics, An Introduction** (2nd ed.), G. Baker and J. Gollub (Cambridge U.P., New York, 1996). A brief introduction to chaos. Emphasizes the driven, damped pendulum and the use of the personal computer. (E,I)

13. **Chaotic and Fractal Dynamics, An Introduction for Applied Scientists and Engineers**, F. C. Moon (Wiley, New York, 1992). An excellent introduction, with an emphasis on engineering applications. (E,I)

14. **From Clocks to Chaos, The Rhythms of Life**, L. Glass and M. C. Mackey (Princeton U.P., Princeton, NJ, 1988). Most of the physiological discussions are accessible to the nonspecialist. (E,I)

15. **Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers**, R. C. Hilborn (Oxford U.P., New York, 1994). A comprehensive introduction to nonlinear dynamics at the sophomore–junior

physics major level. Designed to start from scratch and to bring the reader to the point of being able to deal with the research literature in nonlinear dynamics. (E,I)

16. **Response and Stability**, A. P. Pippard (Cambridge U.P., Cambridge, 1985). Treats driven oscillators, nonlinear oscillators, bifurcations, catastrophes, phase transitions, and broken symmetries at the advanced undergraduate level. (I)
17. **Nonlinear Dynamics and Chaos: With Applications in Physics, Biology, Chemistry and Engineering**, S. H. Strogatz (Addison-Wesley, Reading, MA, 1994). An excellent book for an introductory applied-mathematics course at the undergraduate level. (I)
18. **Chaos: An Introduction to Dynamical Systems**, K. T. Alligood, T. Sauer, and J. A. Yorke (Springer-Verlag, New York, 1996). (I)
19. **Nonlinear Dynamics: A Two-Way Trip from Physics to Math**, H. G. Solari, M. A. Natiello, and G. B. Mindlin (Institute of Physics, Bristol, PA, 1996). (I)
20. **Order within Chaos**, P. Bergé, Y. Pomeau, and C. Vidal (Wiley, New York, 1986). A somewhat dated introduction assuming roughly a first-level graduate-student background in physics. Particularly good discussion of quasi-periodicity and intermittency. (I)
21. **Nonlinear Dynamics and Chaos**, J. M. T. Thompson and H. B. Stewart (Wiley, New York, 1986). Covers a wide range of ''classical'' nonlinear dynamics problems, but there is not much on modern methods such as time-series analysis, generalized dimensions, and so on. (I)
22. **Introduction to Nonlinear Dynamics for Physicists**, H. D. I. Abarbanel, M. I. Rabinovich, and M. M. Sushchik (World Scientific, Singapore, 1993). A compact, but quite useful introduction to nonlinear dynamics at the graduate level. (I)
23. **Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods**, A. H. Nayfeh and B. Balachandran (Wiley, New York, 1995). (I)
24. **Deterministic Chaos, An Introduction** (3rd revised ed.), H. G. Schuster (Wiley, New York, 1995). A rather compact (319 pp.) introduction at roughly the graduate level in physics. (I)
25. **Perspectives of Nonlinear Dynamics** (Vols. 1 and 2), E. A. Jackson (Cambridge U.P., New York, 1989, 1991). A very thoughtful and engaging book. A careful look at mathematical assumptions. Gradually builds up complexity of results, but rather heavy emphasis on analytical methods (perturbation methods, averaging methods, and so on). (I,A)
26. **The Kinematics of Mixing: Stretching, Chaos, and Transport**, J. M. Ottino (Cambridge U.P., Cambridge, 1989). Within the context of fluid flow and transport, this book provides an excellent introduction to chaotic behavior. (I)
27. **An Experimental Approach to Nonlinear Dynamics and Chaos**, N. Tufillaro, T. Abbott, and J. Reilly (Addison-Wesley, Reading, MA, 1992). This book, at the upper-undergraduate and graduate physics level, treats nonlinear dynamics by focusing on experimental systems and using several computer-based models. (I)
28. **Chaotic Dynamics of Nonlinear Systems**, S. N. Rasband (Wiley, New York, 1990). (I)
29. **Chaos in Dynamical Systems**, E. Ott (Cambridge U.P., New York, 1993). A fine book covering many topics in nonlinear dynamics. (I)
30. **Exploring Complexity**, G. Nicolis and I. Prigogine (Freeman, San Francisco, 1989). This wide-ranging book covers many topics in pattern formation, complexity, and chaotic dynamics. Rather compact and scientifically sophisticated. (I,A)

C. Collections of reprints

31. **Chaos**, edited by H. Bai-Lin (World Scientific, Singapore, Vol. I, 1984, Vol. II, 1989). (E,I)
32. **Universality in Chaos** (2nd ed.), edited by P. Cvitanovic (Hilger, Bristol, 1989). (E,I)
33. **Experimental Study and Characterization of Chaos** (Directions in Chaos, Vol. 3), edited by H. Bai-Lin (World Scientific, Singapore, 1990). (E,I)
34. **Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems**, edited by E. Ott, T. Sauer, and J. A. Yorke (Wiley, New York, 1994). Collection of 41 reprints dealing with the analysis of data from nonlinear systems. (I)

D. Advanced texts and monographs

35. **The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors**, C. Sparrow (Springer-Verlag, New York, 1982). (A)
36. **Hamiltonian Dynamical Systems**, R. S. Mackay and J. D. Meiss (Hilger, Bristol, 1987). (A)
37. **Weak Chaos and Quasi-Regular Patterns**, G. M. Zaslavsky, R. Z. Sagdeev, D. A. Usikov, and A. A. Chernikov (Cambridge U.P., Cambridge, 1991). An excellent introduction to stochastic layers and diffusion in the phase space of Hamiltonian systems. (I)
38. **Regular and Chaotic Dynamics** (2nd ed.), A. J. Lichtenberg and M. A. Liebermann (Springer-Verlag, New York, 1992). (A)
39. **Time Series Prediction: Forecasting the Future and Understanding the Past**, edited by A. S. Weigend and N. A. Gershenfeld (Addison-Wesley, Reading, MA, 1993). (I)
40. **Dynamical Systems: Stability, Symbolic Dynamics, and Chaos**, C. Robinson (CRC, Boca Raton, FL, 1994). (A)

E. For the mathematically inclined reader

The following books deal primarily with the mathematical aspects of nonlinear dynamics.

41. **Iterated Maps on the Interval as Dynamical Systems**, P. Collet and J. P. Eckmann (Birkhäuser, Cambridge, MA, 1980). A thorough introduction to the mathematics of iterated maps. (A)
42. **Chaos, Fractals, and Dynamics, Computer Experiments in Mathematics**, R. L. Devaney (Addison-Wesley, Reading, MA, 1990). An introduction (without proofs) to some of the fascinating mathematics of iterated maps, Julia sets, and so on. Accessible to the good secondary school student and most college undergraduates. (E)
43. **A First Course in Chaotic Dynamical Systems**, R. L. Devaney (Addison-Wesley, Reading, MA, 1992). A comprehensive introduction accessible to readers with at least a year of calculus. (E)
44. **An Introduction to Chaotic Dynamical Systems**, R. L. Devaney (Benjamin/Cummings, Menlo Park, CA, 1986). Here are the proofs for the fascinating mathematics of iterated maps. (I)
45. **Encounters with Chaos**, D. Gulick (McGraw-Hill, New York, 1992). This book provides a very readable introduction to the mathematics of one- and two-dimensional iterated map functions with many nice proofs, examples, and exercises. Briefly covers fractals and systems of differential equations. (I)
46. **Dynamics: The Geometry of Behavior**, R. H. Abraham and C. D. Shaw (Addison-Wesley, Reading, MA, 1992). A picture book of chaos! Outstanding diagrams of heteroclinic and homoclinic tangles, and the like. This four-volume series is most useful after you have had some general introduction to chaos. (E,I)
47. **Dynamics and Bifurcations**, J. Hale and H. Koçak (Springer-Verlag, New York, 1991). This book provides a well-thought-out introduction to the mathematics of dynamical systems and bifurcations with many examples. Easily accessible to the advanced undergraduate. (I)
48. **Geometric Methods in the Theory of Ordinary Differential Equations**, V. I. Arnold (Springer-Verlag, New York, 1983). One of the best introductions to the use of geometric methods for nonlinear differential equations. (I)
49. **Differential Equations, Dynamical Systems, and Linear Algebra**, M. W. Hirsch and S. Smale (Academic, New York, 1974). Contains a proof of the famous Poincaré–Bendixson theorem. (A)
50. **Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields** (3rd ed.), J. Guckenheimer and P. Holmes (Springer-Verlag, New York, 1990). A classic in the field, but you need to know your diffeomorphisms from your homeomorphisms. If you are serious about the study of chaos, you will eventually come to this book. (A)

F. Collections of essays

51. **Chaos**, edited by A. V. Holden (Princeton U.P., Princeton, NJ, 1986). A collection of 15 essays by active researchers in the field. (I)
52. **Nonlinearity and Chaos in Engineering Dynamics:** IUTAM Symposium, UCL, July, 1993, edited by J. M. T. Thompson and S. R. Bishop (Wiley, New York, 1994). (I)
53. **Chaotic Evolution and Strange Attractors**, D. Ruelle (Cambridge U.P., New York, 1989). An extended essay (with mathematics) on what the author considers to be the important issues in the statistical analysis (via time series) of chaotic systems. (I)

G. Novels and plays

Chaos and nonlinear dynamics play major roles in the following:

54. **Jurassic Park**, M. Crichton (Ballentine Books, New York, 1990). (E)

55. **Death Qualified, A Mystery of Chaos**, K. Wilhelm (Fawcett Crest, New York, 1991). (E)

56. **Arcadia**, T. Stoppard (Faber and Faber, London, 1993). (E)

V. GENERAL ASPECTS OF NONLINEAR DYNAMICS

A. Historically important books and papers

57. **New Methods of Celestial Mechanics**, H. Poincaré (American Institute of Physics, Woodbury, NY, 1993). An English translation. Where it all begins. (I)

58. ''On relaxation oscillations,'' B. van der Pol, Philos. Mag. (7) **2**, 978–992 (1926). This paper describes the original van der Pol oscillator, a model still used for self-oscillating systems including heart beats. (I)

59. ''Deterministic Nonperiodic Flow,'' E. N. Lorenz, J. Atmos. Sci. **20**, 130–141 (1963). Reprinted in Ref. 31, Vol. I. One of the first modern realizations that nonlinear systems can display sensitive dependence on initial conditions, which then leads to chaotic behavior. (E,I)

60. ''The problem of deducing the climate from the governing equations,'' E. N. Lorenz, Tellus **16** (1), 1–11 (1964). (I)

61. ''Differentiable Dynamical Systems,'' S. Smale, Bull. Am. Math. Soc. **73**, 747–817 (1967). A pioneering paper in the modern theory of dynamical systems. (A)

62. ''On finite limit sets for transformations on the unit interval,'' N. Metropolis, M. L. Stein, and P. R. Stein, J. Combinatorial Theory A **15**, 24–44 (1973). One of the first indications of universal features in nonlinear systems. (I)

63. ''On the Nature of Turbulence,'' D. Ruelle and F. Takens, Commun. Math. Phys. **20**, 167–192 (1971). Reprinted in Ref. 31, Vol. I. This article marks the first appearance of the term strange attractor in the context of nonlinear dynamics. (A)

64. ''Period three implies chaos,'' T.-Y. Li and J. A. Yorke, Am. Math. Monthly **82**, 985–992 (1975). The first appearance of the word ''chaos'' (in its modern technical sense) in the scientific literature. (A)

65. ''Simple Mathematical Models with Very Complicated Dynamics,'' R. M. May, Nature **261**, 459–467 (1976). Reprinted in Refs. 31, Vol. I, and 32. A very influential and quite interesting look at the behavior of iterated map functions, written before the ''discovery'' of Feigenbaum universality. (E)

66. ''The universal metric properties of nonlinear transformations,'' M. Feigenbaum, J. Stat. Phys. **21**, 669–706 (1979). Reprinted in Ref. 31, Vol. I. Provides a proof of the universality of the Feigenbaum parameters α and δ. (I)

67. ''Universal Behavior in Nonlinear Systems,'' M. J. Feigenbaum, Los Alamos Sci. **1**, 4–27 (1980). Reprinted in Ref. 32. Provides a quite readable introduction to the universal features of one-dimensional iterated maps. (E)

B. Survey articles

68. ''Roads to Turbulence in Dissipative Dynamical Systems,'' J.-P. Eckmann, Rev. Mod. Phys. **53**, 643–654 (1981). An early but still useful survey. (I)

69. ''Strange Attractors and Chaotic Motions of Dynamical Systems,'' E. Ott, Rev. Mod. Phys. **53**, 655–672 (1981). (I)

70. ''Chaos,'' J. P. Crutchfield, J. D. Farmer, N. H. Packard, and R. S. Shaw, Sci. Am. **255** (6), 46–57 (1986). A good overview of the field of chaos and its implications for science. Emphasizes the ideas of state space and attractors. (E)

71. ''Classical Chaos,'' R. V. Jensen, Am. Sci. **75**, 168–181 (1987). A well-written, detailed treatment of the major issues in the current study of chaos with an emphasis on mathematics and theory. (E)

72. ''Nonlinearity: Historical and Technological Review,'' R. Landauer, in *Nonlinearity in Condensed Matter*, edited by A. R. Bishop, D. K. Campbell, P. Kumar, and S. E. Trullinger, Springer Series in Solid-State Sciences Vol. 69 (Springer-Verlag, New York, 1987), pp. 3–22. (I)

73. ''Chaos in Deterministic Systems: Strange Attractors, Turbulence, and Applications in Chemical Engineering,'' M. F. Doherty and J. M. Ottino, Chem. Eng. Sci. **43**, 139–183 (1988). A wide-ranging and thoughtful survey of chaos in both dissipative and conservative systems, with an eye on engineering applications, this article is written at roughly the beginning graduate-student level. (I)

74. ''What Is Chaos That We Should Be Mindful of It?'' J. Ford, in **The New Physics**, edited by P. W. Davies (Cambridge U. P., Cambridge, 1989), pp. 348–372. A thoughtful and provocative essay that reviews many of the issues of quantum chaos (as well as many other issues in classical chaos). (E)

75. ''Chaos: A New Scientific Paradigm—or Science by Public Relations,'' M. Dresden, Phys. Teach. **30**, 10–14 and 74–80 (1992). (E)

76. ''Where can one hope to profitably apply the ideas of chaos,'' D. Ruelle, Phys. Today **47** (7), 24–30 (1994). (E)

77. ''Poincaré, Celestial Mechanics, Dynamical-Systems Theory and Chaos,'' P. Holmes, Phys. Rep. **193**, 137–163 (1990). This wide-ranging essay provides insight into the historical development of nonlinear dynamics at a moderately sophisticated mathematical level. (I)

78. ''Chaos with few degrees of freedom,'' M. C. Gutzwiller, Prog. Theor. Phys. Suppl. **116**, 1–16 (1994). (I)

C. Phase space and basins of attraction

Most of the references in Sec. IV B provide introductions to phase space.

79. **Classical Dynamics of Particles and Systems**, J. B. Marion and S. T. Thornton (Harcourt Brace Jovanovic, San Diego, 1988). A nice introduction to phase space and phase diagrams in Chap. 4 in the context of the simple-harmonic oscillator with some discussion of nonlinear oscillators. (I)

Basins of Attraction: For dissipative systems, the set of points that give rise to trajectories landing on a particular attractor are said to lie in that attractor's basin of attraction. The geometry of these basins can be quite complicated. Riddled basins of attraction have the property that any point in the basin has points in another attractor's basin arbitrarily close to it. Their existence implies a further undermining of the goal of predictability—even the long-term attractor cannot be predicted.

80. ''Riddled Basins,'' J. C. Alexander, J. A. Yorke, Z. You, and I. Kan, Int. J. Bifurcations Chaos **2**, 795–813 (1992). (I)

81. ''A physical system with qualitatively uncertain dynamics,'' J. C. Sommerer and E. Ott, Nature (London) **365**, 136–140 (1993). (I)

82. ''The transition to chaotic attractors with riddled basins,'' E. Ott, J. C. Sommerer, J. C. Alexander, I. Kan, and J. A. Yorke, Physica D **76**, 384–410 (1994). (A)

83. ''Experimental and Numerical Evidence for Riddled Basins in Coupled Chaotic Systems,'' J. F. Heagy, T. L. Carroll, and L. M. Pecora, Phys. Rev. Lett. **73**, 3528–3531 (1994). (I)

D. Time series analysis and phase-space reconstruction

84. ''Geometry from a Time Series,'' N. H. Packard, J. P. Crutchfield, J. D. Farmer, and R. S. Shaw, Phys. Rev. Lett. **45**, 712–715 (1980). (I)

85. ''Detecting strange attractors in turbulence,'' F. Takens in **Dynamical Systems and Turbulence**, Lecture Notes in Mathematics Vol. 898, edited by D. A. Rand and L. S. Young (Springer–Verlag, Berlin, 1980). (A)

86. ''Chaos in a noisy world—new methods and evidence from time-series analysis,'' S. Ellner and P. Turchin, Am. Nat. **145** (3), 343–375 (1995). (E)

87. ''State Space Reconstruction in the Presence of Noise,'' M. Casdagli, S. Eubank, J. D. Farmer, and J. Gibson, Physica D **51**, 52–98 (1991). An extensive discussion of state-space reconstruction techniques using embedding. (A)

88. ''Double Poincaré Sections of a Quasi-Periodically Forced, Chaotic Attractor,'' F. C. Moon and W. T. Holmes, Phys. Lett. A **111**, 157–160 (1985). (See also Ref. 13). The Poincaré section technique can be generalized in various ways. For example, in cases where a system is

driven by ''forces'' with two incommensurate frequencies, taking a Poincaré section within a Poincaré section is useful. (I)

1. Periodic orbit analysis

The properties of nonlinear systems can often be determined by looking only at low-order periodic orbits. Poincaré pioneered this method. See Ref. 57, Chap. III, Art. 36.

89. ''Exploring Chaotic Motion Through Periodic Orbits,'' D. Auerbach, P. Cvitanovic, J.-P. Eckmann, G. Gunaratne, and I. Procaccia, Phys. Rev. Lett. **58**, 2387–2389 (1987). (I)

90. ''Progress in the analysis of experimental chaos through periodic orbits,'' R. Badii, E. Brun, M. Finardi, L. Flepp, R. Holzner, J. Parisis, C. Reyl, and J. Simonet, Rev. Mod. Phys. **66** (4), 1389–1415 (1994). (I)

2. Topological analysis

The topology of a state-space attractor tells much about the underlying dynamics.

91. ''Classification of Strange Attractors by Integers,'' G. Mindlin, X.-J. Hou, H. G. Solari, R. Gilmore, and N. B. Tufillaro, Phys. Rev. Lett. **64**, 2350–2353 (1990). (A)

92. ''Relative rotation rates: Fingerprints for strange attractors,'' N. B. Tufillaro, H. G. Solari, and R. Gilmore, Phys. Rev. A **41** (10), 5717–5720 (1990). (A)

93. ''Topological analysis of chaotic time series data from Belousov–Zhabotinski reaction,'' G. B. Mindlin, H. G. Solari, M. A. Natiello, R. Gilmore, and X. J. Hou, J. Nonlinear Sci. **1**, 147–173 (1991). (I)

94. ''Structure of chaos in the laser with saturable absorber,'' F. Papoff, A. Fioretti, E. Arimondo, G. B. Mindlin, H. G. Solari, and R. Gilmore, Phys. Rev. Lett. **68**, 1128–1131 (1992). (I)

95. ''Topological analysis and synthesis of chaotic time series,'' G. B. Mindlin and R. Gilmore, Physica D **58**, 229–242 (1992). Special issue on interpretation of time series from nonlinear dynamics. (I)

96. ''Braids in Classical Dynamics,'' C. Moore, Phys. Rev. Lett. **70**, 3675–3679 (1993). (A)

97. ''Combining Topological Analysis and Symbolic Dynamics to Describe a Strange Attractor and Its Crises,'' M. Lefranc, P. Glorieux, F. Papoff, F. Molesti, and E. Arimondo, Phys. Rev. Lett. **73** (10), 1364–1367 (1994). (I)

98. ''Topological time series analysis of a string experiment and its synchronized model,'' N. B. Tufillaro, P. Wyckoff, R. Brown, T. Schreiber, and T. Molteno, Phys. Rev. E **51** (1), 164–174 (1995). (I)

99. ''Braid analysis of a bouncing ball,'' N. B. Tufillaro, Phys. Rev. E **50** (6), 4509–4522 (1994). (I)

100. ''Structure in the bifurcation diagram of the Duffing oscillator,'' R. Gilmore and J. W. L. McCallum, Phys. Rev. E **51**, 935–956 (1995). (I)

E. Bifurcation theory—routes to chaos, intermittency, and crises

Most of the texts cited in Sec. IV B give introductions to bifurcations and the scenarios that lead from regular behavior to the chaotic behavior. Reference 46 (Part Three, *Global Behavior* and Part Four: *Bifurcation Behavior*) provides lavishly illustrated examples of homoclinic and heteroclinic tangles, including their effects on the Lorenz attractor. Various bifurcation events are depicted graphically. References 47 and 101–102 give a more analytic approach.

101. ''Introduction to Bifurcation Theory,'' J. D. Crawford, Rev. Mod. Phys. **63**, 991–1037 (1991). (I)

102. **Theory of Bifurcations**, V. Afraimovich, V. Arnold, Y. Il'yashenko, and L. Shil'nikov, Dynamical Systems, Vol. 5. Encyclopedia of Mathematical Sciences, edited by V. Arnold (Springer-Verlag, New York, 1993). (A)

1. Period-doubling route to chaos

103. ''From U Sequences to Farey Sequence: A unification of one-parameter scenarios,'' J. Rirgland, N. Issa, and M. Schell, Phys. Rev. A **41**, 4223–4235 (1990). Discusses connections among several routes to chaos. (I)

104. ''Remerging Feigenbaum Trees in Dynamical Systems,'' M. Bier and T. C. Bountis, Phys. Lett. A **104**, 239–244 (1984). This paper discusses ''period bubbling'' and the remerging of the period-doubling cascades observed in iterated maps with more than one parameter. (I)

2. Homoclinic and heteroclinic bifurcations to chaos

When a trajectory asymptotically approaches an unstable fixed point both forwards and backwards in time, the trajectory is said to be homoclinic. If two different fixed points are linked, the trajectory is called heteroclinic. Reference 26, pp. 111–115, provides an excellent illustrated introduction to homoclinic and heteroclinic tangles, which play an important role in chaotic behavior in many systems. Reference 50 provides a detailed formal treatment.

105. ''Homoclinic Chaos,'' edited by P. Gaspard, A. Arneodo, R. Kapral, and C. Sparrow, Physica D **62** (1–4) (1993). Special issue devoted to homoclinic chaos. (A)

3. Intermittency

Behavior that switches, apparently randomly, between periodic and chaotic is called intermittency. It is treated in some detail in the textbooks: Refs. 20, 21, 24. The intermittency route to chaos was introduced by

106. ''Intermittency and the Lorenz Model,'' P. Manneville and Y. Pomeau, Phys. Lett. A **75**, 1–2 (1979). (I)

107. ''Intermittent Transition to Turbulence in Dissipative Dynamical Systems,'' Y. Pomeau and P. Manneville, Commun. Math. Phys. **74**, 189–197 (1980). Reprinted in Refs. 31 (Vol. I) and 32. (A)

In one type of intermittency, called on–off intermittency, the system seems to reside in a quiescent state for varying lengths of time before switching to chaotic behavior.

108. ''On–Off Intermittency: A Mechanism for Bursting,'' N. Platt, E. A. Spiegel, and C. Tresser, Phys. Rev. Lett. **70**, 279–282 (1993). (I)

109. ''Experimental Observation of On–Off Intermittency,'' P. W. Hammer, N. Platt, S. M. Hammel, J. F. Heagy, and B. D. Lee, Phys. Rev. Lett. **73**, 1095–1098 (1994). (I)

110. ''On–Off Intermittency in Spin-Wave Instabilities,'' F. Rödelsperger, A. Cenys, and H. Brenner, Phys. Rev. Lett. **75**, 2594–2597 (1995). (I)

4. Crises

A crisis is the sudden expansion or contraction of a chaotic attractor, another type of bifurcation event.

111. ''Chaotic Attractors in Crisis,'' C. Grebogi, E. Ott, and J. A. Yorke, Phys. Rev. Lett. **48**, 1507–1510 (1982). This paper introduced the concept of a crisis in nonlinear dynamics. (I)

112. ''Crises, Sudden Changes in Chaotic Attractors and Transient Chaos,'' C. Grebogi, E. Ott, and J. A. Yorke, Physica D **7**, 181–200 (1983). (I)

5. Experimental observations of crises

113. ''Intermittent transient chaos at interior crises in the diode resonator,'' R. W. Rollins and E. R. Hunt, Phys. Rev. A **29**, 3327–3334 (1984). (I)

114. ''Quantitative measurement of the parameter dependence of the onset of a crisis in a driven nonlinear oscillator,'' R. C. Hilborn, Phys. Rev. A **31**, 378–382 (1985). (I)

115. ''Laser Chaotic Attractors in Crisis,'' D. Dangoisse, P. Glorieux, and D. Hennequin, Phys. Rev. Lett. **57**, 2657–2660 (1986). (I)

116. ''Experimental Observation of Crisis-Induced Intermittency and Its Critical Exponent,'' W. L. Ditto, S. Rauseo, R. Cawley, C. Grebogi, G.-H. Hsu, E. Kostelich, E. Ott, H. T. Savage, R. Segnan, M. L. Spano, and J. A. Yorke, Phys. Rev. Lett. **63**, 923–926 (1989). (I)

F. Mathematics of dynamical systems

117. ''One-Dimensional Dynamics,'' J. Guckenheimer, Ann. N.Y. Acad. Sci. **357**, 343–347 (1981). Proofs of many properties of iterated map functions and their trajectories. (A)

118. ''A cartoon-assisted proof of Sarkowskii's Theorem,'' H. Kaplan, Am. J. Phys. **55**, 1023–1032 (1987). Sarkowskii's Theorem provides a remarkable ordering of the so-called periodic-windows in a chaotic system. (I)

119. ''First-return maps as a unified renormalization scheme for dynamical systems,'' I. Proccaccia, S. Thomae, and C. Tresser, Phys. Rev. A **35**, 1884–1890 (1987). A unified, but fairly abstract, treatment of very general one-dimensional iterated maps that include the unimodal maps and circle maps as special cases. The dynamic behavior of these general maps is far richer than that of unimodal and circle maps and includes such oddities as period tripling. (A)

120. ''Images of the critical points of nonlinear maps,'' R. V. Jensen and C. R. Myers, Phys. Rev. A **32**, 1222–1224 (1985). Information on the importance of the images of the critical point in organizing patterns of bifurcations. (I)

121. ''Scaling Behavior of Windows in Dissipative Dynamical Systems,'' J. A. Yorke, C. Grebogi, E. Ott, and L. Tedeschini-Lalli, Phys. Rev. Lett. **54**, 1095–1098 (1985). A scaling law for the sizes (as a function of parameter) of the periodic windows of various mappings. (I)

Is the apparently random behavior observed in numerical simulations due to computer round-off and discretization effects? Many things can go wrong when discretizing differential equations.

122. ''Computational chaos—a prelude to computational instability,'' E. N. Lorenz, Physica D **35**, 299–317 (1989). (I)

123. ''Warning—Handle with Care!'' I. Stewart, Nature **355**, 16 (1992). (I)

124. ''Chaos is not an artifact of finite-digit arithmetic,'' R. H. Dalling and M. E. Goggin, Am. J. Phys. **62** (6), 563–564 (1994). (I)

125. ''Comment on 'Chaos is not an artifact of finite-digit arithmetic,' [R. H. Dalling and M. E. Goggin, Am. J. Phys. **62** (6), 563–564 (1994)]','' D. Auerbach, Am. J. Phys. **63** (3), 276 (1995). (I)

126. ''A response to D. Auerbach's Comment,'' M. E. Goggin and R. H. Dalling, Am. J. Phys. **63** (3), 277 (1995). (I)

127. ''Shadowing of Physical Trajectories in Chaotic Dynamics: Containment and Refinement,'' C. Grebogi, S. M. Hammel, J. A. Yorke, and T. Sauer, Phys. Rev. Lett. **65**, 1527–1530 (1990). Even though the computed trajectory may not be the one you desired, it does follow *some* actual trajectory. (I)

G. Predictability, determinism, the butterfly effect, and other philosophical issues

128. ''The Recently Recognized Failure of Predictability in Newtonian Dynamics,'' J. Lighthill, Proc. R. Soc. London, Ser. A **407**, 35–50 (1986). (E)

129. **In the Wake of Chaos**, S. H. Kellert (The University of Chicago Press, Chicago, 1993). A philosophical analysis of the many fundamental questions raised by chaos theory. (E,I)

H. Quasi-periodicity and number theory

130. ''Onset of Turbulence in a Rotating Fluid,'' J. P. Gollub and H. L. Swinney, Phys. Rev. Lett. **35**, 927–930 (1975). The first experimental evidence for the Ruelle–Takens quasiperiodic route to chaos. A pioneering paper in the contemporary development of nonlinear dynamics. (I)

131. ''Scaling Behavior in a Map of a Circle onto Itself: Empirical Results,'' S. Shenker, Physica D **5**, 405–411 (1982). Reprinted in Ref. 32. (I)

132. ''Quasiperiodicity in Dissipative Systems: A Renormalization Group Analysis,'' M. J. Feigenbaum, L. P. Kadanoff, and S. J. Shenker, Physica D **5**, 370–386 (1982). Reprinted in Ref. 31, Vol. I. (I)

133. ''Universal Transition from Quasi-Periodicity to Chaos in Dissipative Systems,'' D. Rand, S. Ostlund, J. Sethna, and E. Siggia, Phys. Rev. Lett. **49**, 132–135 (1982). Reprinted in Ref. 31, Vol. I. (I)

134. ''Renormalization, Unstable Manifolds, and the Fractal Structure of Mode Locking,'' P. Cvitanovic, M. H. Jensen, L. P. Kadanoff, and I. Procaccia, Phys. Rev. Lett. **55**, 343–346 (1985). (A)

135. ''Universality in the quasiperiodic route to chaos,'' T. W. Dixon, T. Gherghetta, and B. G. Kenny, Chaos **6** (1), 32–42 (1996). A good survey of the universal features of the quasi-periodic route to chaos illustrated with numerical results. (I)

Number theory plays a surprisingly important role in dynamics. See Ref. 15, pp. 272–280. Some easily accessible introductions to number theory are

136. **The Divine Proportion**, H. E. Huntley (Dover, New York, 1970). A delightful source of information on the Golden Mean. (E)

137. **The Theory of Numbers**, G. H. Hardy and E. M. Wright (Oxford U.P., Oxford, 1938). (I)

I. Hamiltonian chaos

Hamiltonian dynamics deals with systems in which dissipation is negligible. There are no attracting sets. Chaotic behavior and periodic behavior may occur for different initial conditions for the same set of parameters.

138. ''Self-Generated Chaotic Behavior in Nonlinear Mechanics,'' R. H. G. Helleman, in **Fundamental Problems in Statistical Mechanics**, Vol. 5. edited by E. G. D. Cohen, (North-Holland, Amsterdam, 1980), pp. 165–233. Reprinted in Ref. 32. A superb introduction to the chaotic behavior of nonintegrable Hamiltonian systems. (I)

139. ''Chaos: How Regular Can It Be?'' A. Chernikov, R. Sagdeev, and G. Zaslavsky, Phys. Today **41** (11), 27–35 (1988). A nice overview of the question of mixing and chaotic behavior in Hamiltonian systems. (E,I)

140. ''Hamiltonian Chaos,'' N. Srivastava, C. Kaufman, and G. Müller, Comput. Phys. **4**, 549–553 (1990) and ''Hamiltonian Chaos II,'' **5**, 239–243 (1991). A brief but insightful introduction to Hamiltonian chaos for someone with an undergraduate background in classical mechanics. (I)

1. Period doubling in Hamiltonian systems

141. ''Universal Behavior in Families of Area-Preserving Maps,'' J. M. Greene, R. S. MacKay, F. Vivaldi, and M. J. Feigenbaum, Physica D **3**, 468–486 (1981). (A)

142. ''Period Doubling Bifurcations and Universality in Conservative Systems,'' T. C. Bountis, Physica D **3**, 577–589 (1981). (I)

2. KAM theorem and related topics

The famous Kolmogorov–Arnold–Moser theorem describes how periodic and quasi-period orbits change to chaotic ones in Hamiltonian systems. For an introduction see Ref. 50, pp. 219–223, and Ref. 25, Vol. 2, Appendix L.

143. **Mathematical Methods in Classical Mechanics**, V. I. Arnold (Springer-Verlag, New York, 1978). (I)

144. ''A Universal Instability of Many Dimensional Oscillator Systems,'' B. V. Chirikov, Phys. Rep. **52**, 263–379 (1979). The Chirikov Standard Map is discussed in this lengthy review of Hamiltonian dynamics as manifest in nonlinear oscillators. (A)

145. ''Dissipative Standard Map,'' G. Schmidt and B. H. Wang, Phys. Rev. A **32**, 2994–2999 (1985). Explores the connection between dissipative and Hamiltonian systems. (I)

J. Transient chaos and chaotic scattering

Many systems exhibit chaotic behavior as a transient effect before settling down to periodic behavior.

146. ''Transient Chaos,'' T. Tél, in Ref. 33, pp. 149–211. (I)

147. ''Exploring transient chaos in an NMR-laser experiment,'' I. M. Jánosi, L. Flepp, and T. Tél, Phys. Rev. Lett. **73** (4), 529–532 (1994). (I)

148. ''Time-Series Analysis of Transient Chaos,'' I. M. Jánosi and T. Tél, Phys. Rev. E **49** (4), 2756–2763 (1994). (I)

Chaotic scattering: Spatial and temporal behavior also become linked in the study of scattering problems. The basic idea is that when a moving object collides with another object and is deflected by it, the scattering process may show sensitive dependence to the details of the incoming trajectory, another kind of sensitive dependence on initial conditions, resulting in ''chaotic scattering.'' For some taste of this field see

149. ''Irregular Scattering,'' B. Eckhardt, Physica D **33**, 89–98 (1988). (I)
150. ''Experimental Demonstration of Chaotic Scattering of Microwaves,'' E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072–3075 (1990). (I)
151. ''Chaotic scattering: An introduction,'' E. Ott and T. Tél, Chaos **3** (4), 417–426 (1993). Lead-off article for focus issue on chaotic scattering. (I)
152. ''Chaotic Scattering,'' T. Yalcinkaya and Y. C. Lai, Comput. Phys. **9** (5), 511–518 (1995). (I)

K. Quantifying chaos—Lyapunov exponents, entropies, and generalized dimensions.

1. General references on quantifying chaos

See Ref. 34.

153. ''An Experimentalist's Introduction to the Observation of Dynamical Systems,'' N. Gershenfeld in Ref. 31, Vol. II, pp. 310–382. An excellent introduction to many techniques for quantifying chaos with an eye on experimental data. (E)
154. ''The analysis of observed data in physical systems,'' H. D. I. Abarbanel, R. Brown, J. J. Sidorowich, and L. Sh. Tsimring, Rev. Mod. Phys. **65**, 1331–1392 (1993). A comprehensive survey of methods of analyzing data suspected of exhibiting chaotic behavior. (I)
155. ''Ergodic Theory of Chaos and Strange Attractors,'' J.-P. Eckmann and D. Ruelle, Rev. Mod. Phys. **57**, 617–656 (1985). A review article surveying many methods of quantifying chaos at a more sophisticated level. (A)
156. **Analysis of Observed Chaotic Data**, H. D. I. Abarbanel (Springer-Verlag, New York, 1996). (I)

2. Surrogate data

To distinguish chaotic behavior from noisy behavior, it is useful to generate surrogate data from your original data set.

157. ''Do climatic attractors exist?'' P. Grassberger, Nature **323**, 609–612 (1986). (I)
158. ''Testing for nonlinearity in time series: The method of surrogate data,'' J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, and J. D. Farmer, Physica D **58**, 77–94 (1992). (I)

3. Lyapunov exponents

Lyapunov exponents quantify the exponential separation of nearby trajectories, the hallmark of chaotic behavior.

159. ''Liapunov Exponents from Time Series,'' J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, and S. Ciliberto, Phys. Rev. A **34**, 4971–4979 (1986). (I)
160. ''Quantifying Chaos with Lyapunov Exponents,'' A. Wolf, in Ref. 51, pp. 273–290. (I)
161. ''Determining Lyapunov Exponents from a Time Series,'' A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vasano, Physica D **7**, 285–317 (1985). A widely used method for determining Lyapunov exponents directly from data. (I)
162. ''Studying Chaotic Systems Using Microcomputer Simulations and Lyapunov Exponents,'' S. DeSouza-Machado, R. W. Rollins, D. T. Jacobs, and J. L. Hartman, Am. J. Phys. **58**, 321–329 (1990). A good introduction to Lyapunov exponents and their calculation in the case when the time-evolution equations are known. (I)
163. ''Lyapunov exponents for pedestrians,'' J. C. Earnshaw and D. Haughey, Am. J. Phys. **61** (5), 401–407 (1993). (E)
164. ''Estimation of Lyapunov exponents from time series: The stochastic case,'' M. Dämmig and F. Mitschke, Phys. Lett. A **178**, 385–394 (1993). The scheme in Ref. 161 for Lyapunov exponents has problems. (I)
165. ''A practical method for calculating largest Lyapunov exponents from small data sets,'' M. T. Rosenstein, J. J. Collins, and C. J. De Luca, Physica D **65**, 117–134 (1993). (I)

4. Scaling law for the average Lyapunov exponent

166. ''Scaling Behavior of Chaotic Flows,'' B. A. Huberman and J. Rudnick, Phys. Rev. Lett. **45**, 154–156 (1980). (I)
167. ''Experimental Verification of a Universal Scaling Law for the Lyapunov Exponent of a Chaotic System,'' S. C. Johnston and R. C. Hilborn, Phys. Rev. A **37**, 2680–2182 (1988). (I)

5. Kolmogorov entropy

An entropy-like quantity can also be used to characterize chaotic behavior.

168. ''Information Dimension and Probabilistic Structure of Chaos,'' D. Farmer, Z. Naturforsch. **37a**, 1304–1325 (1982). (I)
169. ''Estimation of the Kolmogorov Entropy from a Chaotic Signal,'' P. Grassberger and I. Procaccia, Phys. Rev. A **28**, 2591–2593 (1983). (I)
170. ''Kolmogorov Entropy and Numerical Experiments,'' G. Benettin, L. Galgani, and J.-M. Strelcyn, Phys. Rev. A **14**, 2338–2345 (1976). Reprinted in Ref. 31, Vol. I. A nice discussion of the relationship between K-entropy and Lyapunov exponents. (I)

6. Attractor dimensions

If the ''dimension'' of a phase-space attractor is not an integer, then the attractor is said to be a strange attractor. The corresponding dynamical behavior is usually chaotic. A good survey and comparison of the various kinds of dimensions can be found in Ref. 13.

171. ''The Dimension of Chaotic Attractors,'' D. Farmer, E. Ott, and J. A. Yorke, Physica D **7**, 153–180 (1983). Careful definitions of Hausdorff dimension, capacity (box-counting) dimension, and information dimension with various examples. (E)
172. ''Fractal Dimension: Limit Capacity or Hausdorff Dimension?'' C. Essex and M. Nerenberg, Am. J. Phys. **58**, 986–988 (1990). Some examples of sets that yield different numerical values for the different definitions of dimensions. (I)

7. Generalized dimensions

To completely characterize a chaotic attractor, you must use an infinity of generalized dimensions. For an elementary introduction, see Ref. 15.

173. ''Generalized Dimensions of Strange Attractors,'' P. Grassberger, Phys. Lett. A **97**, 227–230 (1983). (I)
174. ''The Infinite Number of Generalized Dimensions of Fractals and Strange Attractors,'' H. Hentschel and I. Procaccia, Physica D **8**, 435–444 (1983). (I)

8. Correlation dimension

The correlation dimension is by far the most widely used method of characterizing strange attractors.

175. ''Characterization of Strange Attractors,'' P. Grassberger and I. Procaccia, Phys. Rev. Lett. **50**, 346–349 (1983). The introduction of the correlation dimension. (I)
176. ''Strange Attractors in Weakly Turbulent Couette–Taylor Flow,'' A. Brandstater and H. L. Swinney, Phys. Rev. A **35**, 2207–2220 (1987). This paper analyzes experimental data using the embedding technique and carefully examines how choices of time lags, sampling rates, embedding dimensions, and number of data points affect the computed value of the correlation dimension. (I)

177. ''Calculating the Dimension of Attractors from Small Data Sets,'' N. B. Abraham, A. M. Albano, B. Das, G. De Guzman, S. Yong, R. S. Gioggia, G. P. Puccioni, and J. R. Tredicce, Phys. Lett. A **114**, 217–221 (1986). Good discussion of the computational pitfalls surrounding correlation dimensions. (I)

178. ''Spurious Dimension from Correlation Algorithms Applied to Limited Time-Series Data,'' J. Theiler, Phys. Rev. A **34**, 2427–2432 (1986). (I)

179. ''Singular-Value Decomposition and the Grassberger–Procaccia Algorithm,'' A. M. Albano, J. Muench, C. Schwartz, A. I. Mees, and P. E. Rapp, Phys. Rev. A **38**, 3017–3026 (1988). This paper suggests the use of the so-called singular-value decomposition to enhance the speed of calculation of the correlation dimension. (I)

180. ''Deterministic Chaos: The Science and the Fiction,'' D. Ruelle, Proc. R. Soc. London, Ser. A **427**, 241–248 (1990). This essay includes cautions on calculating the correlation dimension with small data sets. (I)

181. ''Estimating correlation dimension from a chaotic time series: When does plateau onset occur?'' M. Ding, C. Grebogi, E. Ott, T. Sauer, and J. A. Yorke, Physica D **69**, 404–424 (1993). Some important details in determining correlation dimension from actual data sets. (I)

182. ''Optimal Reconstruction Space for Estimating Correlation Dimension,'' R. C. Hilborn and M. Ding, Int. J. Bifurcations Chaos **6** (2), 377–381 (1996). A way to estimate the ''best'' embedding dimension for determining the correlation dimension. (I)

9. Kaplan–Yorke conjecture

There is a connection between Lyapunov exponents characterizing the temporal divergence of phase-space trajectories and the ''dimensions'' that quantify the geometric character of phase-space attractors.

183. ''Chaotic Behavior of Multidimensional Difference Equations,'' J. L. Kaplan and J. A. Yorke, in **Functional Differential Equations and Approximations of Fixed Points**, Springer Lecture Notes in Mathematics Vol. 730, edited by H.-O. Peitgen and H.-O. Walter (Springer-Verlag, Berlin, 1979), pp. 204–240. (I)

L. Fractals, multifractals, and the thermodynamic formalism

The geometry of strange attractors is best described by fractals.

184. ''Resource Letter FR-1: Fractals,'' A. J. Hurd, Am. J. Phys. **56** (11), 969–975 (1989). (I)

The following books and article provide excellent introductions to fractal geometry:

185. The Fractal Geometry of Nature, B. B. Mandlebrot (Freeman, San Francisco, 1982). The masterful (but often frustratingly diffusive) account by the inventor of the term fractal. (E,I)

186. The Beauty of Fractals, H.-O. Peitgen and P. H. Richter (Springer-Verlag, Berlin, 1986). A book with beautiful pictures and enough detail of the mathematics so you can begin developing your own fractal pictures. (I)

187. Fractals Everywhere, M. Barnsley (Academic, San Diego, 1988). A more mathematically sophisticated introduction to fractals. (A)

188. ''The Language of Fractals,'' H. Jurgens, H.-O. Peitgen, and D. Saupe, Sci. Am. **263** (2), 60–67 (1990). (E)

1. Fractal basin boundaries

189. ''Intermittent Chaos and Low-Frequency Noise in the Driven Damped Pendulum,'' E. G. Gwinn and R. M. Westervelt, Phys. Rev. Lett. **54**, 1613–1616 (1985). (I)

190. ''Fractal Basin Boundaries and Intermittency in the Driven Damped Pendulum,'' E. G. Gwinn and R. M. Westervelt, Phys. Rev. A **33**, 4143–4155 (1986). (I)

2. Multifractals and $f(\alpha)$

Most attractors in nonlinear dynamics are not simple fractals, but require generalized fractal dimensions to describe them completely. $f(\alpha)$ gives a statistical distribution of fractal dimensions.

191. ''Fractal Measures and Their Singularities: The Characterization of Strange Sets,'' T. C. Halsey, M. H. Jensen, L. P. Kadanoff, I. Procaccia, and B. I. Schraiman, Phys. Rev. A **33**, 1141–1151 (1986). (A)

192. ''Direct Determination of the $f(\alpha)$ Singularity Spectrum,'' A. Chhabra and R. V. Jensen, Phys. Rev. Lett. **62**, 1327–1330 (1989). (A)

193. ''Multifractal Phenomena in Physics and Chemistry,'' H. E. Stanley and P. Meakin, Nature **335**, 405–409 (1988). (E,I)

3. Generalized entropies, $g(\Lambda)$, *and the thermodynamic formalism*

The Kolmogorov entropy and its generalizations are closely related to Lyapunov exponents. For an introduction, see Ref. 15.

194. Probability Theory, A. Renyi (North-Holland, Amsterdam, 1970). (I)

195. ''Estimation of the Kolmogorov Entropy from a Chaotic Signal,'' P. Grassberger and I. Procaccia, Phys. Rev. A **28**, 2591–2593 (1983). The connection between the derivative of the correlation sum with embedding dimension and the K entropy. (I)

196. ''Dimensions and Entropies of Strange Attractors from a Fluctuating Dynamics Approach,'' P. Grassberger and I. Procaccia, Physica D **13**, 34–54 (1984). (A)

197. ''Dynamical Spectrum and Thermodynamic Functions of Strange Sets from an Eigenvalue Problem,'' T. Tél, Phys. Rev. A **36**, 2507–2510 (1987). (A)

198. ''Generalized Dimensions and Entropies from a Measured Time Series,'' K. Pawelzik and H. G. Schuster, Phys. Rev. A **35**, 481–484 (1987). Discussion of the generalized correlation sums C_q^d and $g(\Lambda)$ distribution. (A)

199. ''The Thermodynamics of Fractals,'' T. Bohr and T. Tél, in Ref. 31, Vol. II. A thorough discussion of the thermodynamic formalism (but limited to one-dimensional systems). (A)

M. Noise and stochastic resonance

How does noise, present in all real systems, affect nonlinear dynamics? Can nonlinear methods be used to reduce the effects of noise?

200. ''Fluctuations and simple chaotic dynamics,'' J. P. Crutchfield, J. D. Farmer, and B. A. Huberman, Phys. Rep. **92**, 45–82 (1982). (I)

201. ''Scaling for external noise at the onset of chaos,'' J. Crutchfield, M. Nauenberg, and J. Rudnick, Phys. Rev. Lett. **46**, 933–935 (1981). (I)

202. ''Functional renormalization group theory of universal $1/f$-noise in dynamical systems,'' I. Procaccia and H. G. Schuster, Phys. Rev. A **28**, 1210–1212 (1983). (A)

203. ''Scaling law for characteristic times of noise-induced crises,'' J. C. Sommerer, E. Ott, and C. Grebogi, Phys. Rev. A **43**, 1754–1769 (1991). The theory of so-called noise-induced crises. (I)

204. ''Experimental Confirmation of the Scaling Theory of Noise-Induced Crises,'' J. C. Sommerer, W. L. Ditto, C. Grebogi, E. Ott, and M. L. Spano, Phys. Rev. Lett. **66**, 1947–1950 (1991). (I)

1. Noise-reduction methods

205. ''On noise reduction methods for chaotic data,'' P. Grassberger, R. Hegger, H. Kantz, C. Schaffrath, and Th. Schreiber, Chaos **3**, 127–140 (1993). Compares various ways of reducing the effects of noise in real data using nonlinear dynamics techniques. (I)

206. ''Noise reduction in chaotic time-series data: A survey of common methods,'' E. J. Kostelich and Th. Schreiber, Phys. Rev. E **48**, 1752–1762 (1993). (I)

207. ''Nonlinear noise reduction—A case study on experimental data,'' H. Kantz, Th. Schreiber, I. Hoffman, T. Buzug, G. Pfister, L. G. Flepp, J. Simonet, R. Badii, and E. Brun, Phys. Rev. E **48** (2), 1529–1538 (1993). (I)

2. *Stochastic resonance*

Adding more noise paradoxically can increase the signal-to-noise ratio under certain conditions.

208. ''Tuning in to Noise,'' A. R. Bulsara and L. Gammaitoni, Phys. Today **49** (3), 39–45 (1996). (E)

209. ''The Benefits of Background Noise,'' F. Moss and K. Wiesenfeld, Sci. Am. **273** (2), 66–69 (1995). (E)

210. ''Stochastic resonance and the benefits of noise: From ice ages to crayfish and squids,'' K. Wiesenfeld and F. Moss, Nature **373**, 33–36 (1995). (I)

N. Hysteresis

211. ''Scaling Laws for Dynamical Hysteresis in a Multidimensional Laser System,'' A. Hohl, H. J. C. van der Linden, R. Roy, G. Goldsztein, F. Broner, and S. H. Strogatz, Phys. Rev. Lett. **74**, 2220–2223 (1995). (I)

VI. APPLICATIONS

A. Simple mechanical systems, engineering applications, acoustics

Reference 98 contains an analysis of a vibrating string experiment.

212. ''Horseshoes in the driven, damped pendulum,'' E. G. Gwinn and R. M. Westervelt, Physica D **23**, 396–401 (1986). (I)

213. ''Chaotic dynamics of a bouncing ball,'' N. B. Tufillaro and A. M. Albano, Am. J. Phys. **54** (10), 939–944 (1986). (I)

214. ''Nonlinear and Chaotic String Vibrations,'' N. B. Tufillaro, Am. J. Phys. **57**, 408–414 (1989). (I)

215. ''Two balls in one dimension with gravity,'' N. D. Whelan, D. A. Goodings, and J. K. Cannizzo, Phys. Rev. A **42**, 742–754 (1990). (I)

216. ''Impact Oscillators,'' S. R. Bishop, Philos. Trans. R. Soc. London (Phys. Sci. Eng.) **347** (1683), 347–351 (1993). (I)

217. ''Stability and Hopf bifurcations in an inverted pendulum,'' J. A. Blackburn, H. J. T. Smith, and N. Gronbechjensen, Am. J. Phys. **61** (10), 903–908 (1993). (I)

218. ''Chaos in a double pendulum,'' T. Shinbrot, C. Grebogi, J. Wisdom, and J. A. Yorke, Am. J. Phys. **60** (6), 491–499 (1992). (I)

219. ''Deterministic chaos in the elastic pendulum—a simple laboratory for nonlinear dynamics,'' R. Cuerno, A. F. Ranada, and J. J. Ruizlorenzo, Am. J. Phys. **60** (1), 73–79 (1992). (I)

220. ''Double pendulum—an experiment in chaos,'' R. B. Levien, and S. M. Tan, Am. J. Phys. **61** (11), 1038–1044 (1993). (I)

221. ''Chaos at the amusement park—dynamics of the tilt-a-whirl,'' R. L. Kautz and B. M. Huggard, Am. J. Phys. **62** (1), 59–66 (1994). (I)

222. ''Chaos in a simple impact oscillator: The Bender bouncer,'' J. S. Walker and T. Soule, Am. J. Phys. **64** (4), 397–409 (1996). A nice example of a ''piecewise linear'' system and how chaos can arise from boundary conditions; nonlinearity in the evolution equations is not required. (I)

Acoustics

223. ''Methods of chaos physics and their application to acoustics,'' W. Lauterborn and U. Parlitz, J. Acoust. Soc. Am. **84**, 1975–1993 (1988). (I)

224. ''Introduction to acoustical chaos,'' D. Crighton and L. A. Ostrovsky, Chaos **5** (3), 495 (1995). Introduction to a focus issue on acoustical chaos. (I)

B. Simple electronic systems

225. ''Evidence for Universal Chaotic Behavior of a Driven Nonlinear Oscillator,'' J. Testa, J. Perez, and C. Jeffries, Phys. Rev. Lett. **48**, 714–717 (1982). [See the Comment on this article by E. R. Hunt, Phys. Rev. Lett. **49**, 1054 (1982).] (I)

226. ''Exactly Solvable Model of a Physical System Exhibiting Universal Chaotic Behavior,'' R. Rollins and E. R. Hunt, Phys. Rev. Lett. **49**, 1295–1298 (1982). (I)

227. ''Simplest Chaotic Nonautonomous Circuit,'' T. Matsumoto, L. O. Chua, and S. Tanaka, Phys. Rev. A **30**, 1155–1157 (1984). (I)

228. ''Observation of Chaotic Dynamics of Coupled Nonlinear Oscillators,'' R. Van Buskirk and C. Jeffries, Phys. Rev. A **31**, 3332–3357 (1985). A detailed description of how one can study chaos using simple semiconductor diodes. Lots of pictures and diagrams. (I)

229. ''Universal Scaling and Chaotic Behavior of a Josephson-Junction Analog,'' W. J. Yeh and Y. H. Kao, Phys. Rev. Lett. **49**, 1888–1891 (1982). (I)

230. ''Type-II intermittency in a coupled nonlinear oscillator: Experimental observation,'' J.-Y. Huang and J.-J. Kim, Phys. Rev. A **36**, 1495–1497 (1987). (I)

231. ''The Electronic Bouncing Ball,'' R. L. Zimmerman, S. Celaschi, and L. G. Neto, Am. J. Phys. **60** (4), 370–375 (1992). (I)

232. ''How to Generate Chaos at Home,'' D. Smith, Sci. Am. **266** (1), 144–146 (1992). (E)

233. ''The Chaotic Oscilloscope,'' M. T. Levinsen, Am J. Phys. **61** (2), 155–165 (1993). An oscilloscope combined with a photodiode shows chaotic behavior. (I)

234. Special issue on chaos in nonlinear electronic circuits, L. O. Chua and M. Hasler, IEEE Trans. Circuits Syst. **40** (10-11) (1993). (I)

C. Biological and medical systems

235. **The Geometry of Biological Time**, A. T. Winfree (Springer-Verlag, New York, 1980). (E,I)

236. ''Global Bifurcations and Chaos in a Periodically Forced Biological Oscillator,'' L. Glass, M. R. Guevar, J. Belair, and A. Shrier, Phys. Rev. A **29**, 1348–1357 (1984). (I)

237. ''Chaotic Cardiac Rhythms,'' L. Glass, A. Shrier, and J. Belair, in Ref. 51, pp. 237–256. (I)

238. ''A Circle Map in a Human Heart,'' M. Courtemancho, L. Glass, J. Belari, D. Scagliotti, and D. Gordon, Physica D **40**, 299–310 (1989). (I)

239. ''How brains make chaos in order to make sense of the world,'' C. A. Skarda and W. J. Freeman, Behav. Brain Sci. **10**, 161–195 (1987), with comments from respondents. (I)

240. ''Cardiac arrhythmias and circle maps—A classical problem,'' L. Glass, Chaos **1** (1), 13–19 (1991). (I)

241. ''Nonlinear dynamics of physiological function and control,'' L. Glass, Chaos **1** (3), 247–250 (1991). Lead-off article for a focus issue on nonlinear dynamics of physiological control. (I)

242. ''Coupled Oscillators and Biological Synchronization,'' S. Strogatz and I. Stewart, Sci. Am. **269** (6), 102–109 (1993). (E)

243. ''Why chaos is best for hearts and minds,'' R. Pease, New Sci. **146** (1982), 18 (1995). (E)

244. ''Dynamical disease: Identification, temporal aspects and treatment strategies,'' J. Bélair, L. Glass, U. an der Heiden, and J. Milton, Chaos **5** (1), 1–7 (1995). Lead-off article for focus issue on dynamical disease: mathematical analysis of human illness. (I)

D. Chemical reaction systems

245. ''One-dimensional Dynamics in a Multicomponent Chemical Reaction,'' R. H. Simoyi, A. Wolf, and H. L. Swinney, Phys. Rev. Lett. **49**, 245–248 (1982). (I)

246. ''Multiplicity in a Chemical Reaction with One-Dimensional Dynamics,'' K. Coffman, W. D. McCormick, and H. L. Swinney, Phys. Rev. Lett. **56**, 999–1002 (1986). A quite readable description of the observations of violations of the MSS *U*-sequence [Ref. 62] for periodic windows in an experimental study of oscillating chemical reactions. (I)

247. ''Universality, multiplicity, and the effect of iron impurities in the Belousov–Zhabotinskii reaction,'' K. G. Coffman, W. D. McCormick, R. H. Noszticziu, Z. Simoyi, and H. L. Swinney, J. Chem. Phys. **86**, 119–129 (1987). (I)

248. **Oscillations, Waves, and Chaos in Chemical Kinetics**, S. K. Scott (Oxford U. P., New York, 1994). (I)

Reaction–diffusion systems

249. ''The Chemical Basis of Morphogenesis,'' A. M. Turing, Philos. Trans. R. Soc. London, Ser. B **237**, 37–72 (1952). (I)

250. ''Bifurcation, Propagating Solutions, and Phase Transitions in a Nonlinear Reaction with Diffusion,'' A. M. Albano, N. B. Abraham, D. E. Chyba, and M. Martelli, Am. J. Phys. **52**, 161–167 (1984). (I)

251. ''Regular and Chaotic Chemical Spatiotemporal Patterns,'' W. Y. Tam, J. A. Vastano, H. L. Swinney, and W. Horsthemke, Phys. Rev. Lett. **61**, 2163–2166 (1988). (I)
252. ''A history of chemical oscillations and waves,'' A. M. Zhabotinsky, Chaos **1** (4), 379–386 (1991). Lead-off article for focus issue on nonlinear dynamics in chemistry. (I)
253. ''Transition to Chemical Turbulence,'' Q. Ouyang and H. L. Swinney, Chaos **1**, 411–419 (1991). (I)

E. Lasers and other optical systems

254. ''Evidence for Lorenz-Type Chaos in a Laser,'' C. O. Weiss and J. Brock, Phys. Rev. Lett. **57** (22), 2804–2806 (1986). (I)
255. ''Optical Chaos,'' P. W. Milonni, J. R. Ackerhalt, and M.-L. Shih, Optics News 34–37 (March, 1987). (I)
256. ''Complexities and progress in studies of optical chaos,'' Optics News 8–12 (August, 1989). (I)
257. ''Homoclinic and Heteroclinic Chaos in a Single-Mode Laser,'' C. O. Weiss, N. B. Abraham, and U. Hübner, Phys. Rev. Lett. **61** (14), 1587–1590 (1988). (I).

Two issues of the Journal of the Optical Society of America have been devoted to nonlinear dynamics in optics:

258. ''Overview of Instabilities in Laser Systems,'' N. B. Abraham, L. A. Lugiato, and L. M. Narducci, J. Opt. Soc. Am. B **2**, 7–13 (1985). This lead-off article for the issue devoted to ''Instabilities in Active Optical Media'' gives a good survey of the field. (I)
259. ''Nonlinear Dynamics of Lasers,'' J. Opt. Soc. Am. B **5** (5) (May, 1988). (I)
260. **Laser Physics and Laser Instabilities**, L. M. Narducci and N. B. Abraham (World Scientific, Singapore, 1988). (I)
261. ''Intermittency in the Coherence Collapse of a Semiconductor Laser with External Feedback,'' J. Sacher, W. Elsässer, and E. Göbel, Phys. Rev. Lett. **63**, 2224–2227 (1989). (I)
262. ''Experimental Evidence of Chaotic Itinerancy and Spatiotemporal Chaos in Optics,'' F. T. Arecchi, G. Giacomelli, P. L. Ramazza, and S. Residori, Phys. Rev. Lett. **65** (20), 2531–2534 (1990). (I)
263. ''Type III intermittency of a laser,'' D. Y. Tang, J. Pujol, and C. O. Weiss, Phys. Rev. A **44**, 35–38 (1991). (I)
264. ''Chaotic behavior in transverse-mode laser dynamics,'' W. Kaige, N. B. Abraham, and A. M. Albano, Chaos **3** (3), 287–294 (1993). (I)
265. ''Flowerlike Patterns Generated by a Laser Beam Transmitted through a Rubidium Cell with Single Feedback Mirror,'' G. Grynberg, A. Maître, and A. Petrossian, Phys. Rev. Lett. **72** (15), 2379–2382 (1994). (I)

F. Pattern formation and spatiotemporal chaos

Nonlinear wave effects are important in many areas of physics.

266. **Nonlinear Waves, Solitons and Chaos**, E. Infeld and G. Rowlands (Cambridge, U. P., Cambridge, 1990). (I)

Nonlinear systems can generate interesting spatial patterns and the dynamics can be much more complicated when spatial degrees of freedom are included. (For examples in optics, see Sec. VI E.)

1. General introductions to pattern formation

267. ''Self-Organizing Structures,'' B. F. Madure and W. L. Freedman, Am. Sci. **75**, 252–259 (1987). (E)
268. **Mathematical Biology**, J. D. Murray (Springer-Verlag, New York, 1989). An extensive treatment of the mathematics of pattern formation (and many other topics) in biology. (I)
269. ''Synergetics: An Overview,'' H. Haken, Rep. Prog. Phys. **52**, 515-533 (1989). (I)
270. ''Disorder, Dynamical Chaos, and Structures,'' A. V. Gaponov-Grekhov and M. I. Rabinovich, Phys. Today **43** (7), 30–38 (July, 1990). (I)
271. **Origins of Order: Self-Organization and Selection in Evolution**, S. A. Kaufman (Oxford U. P., New York, 1991). (I)
272. ''Pattern-Formation Outside of Equilibrium,'' M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65** (3), 851–1112 (1993). A comprehensive survey of pattern formation in nonlinear systems. (I)
273. ''Nature's Algorithms,'' B. Hayes, Am. Sci. **82**, 206–210 (1994). Nice introductory survey of various kinds of pattern growth. (E)
274. ''Spatio-temporal patterns: Observation and analysis,'' H. L. Swinney, in Ref. 39, pp. 557–567. (I)
275. ''Spatiotemporal Chaos.'' M. C. Cross and P. C. Hohenberg, Science **263** (5153), 1569–1570 (1994). (E)
276. ''Oscillating granular layers produce stripes, squares, hexagons...,'' G. B. Lubkin, Phys. Today **48** (10), 17–19 (1995).(E)

2. Coupled iterated maps and coupled oscillator models

277. ''Pattern Dynamics in Spatiotemporal Chaos,'' K. Kaneko, Physica D **34**, 1–41 (1989). (I)
278. ''Overview of coupled map lattices,'' K. Kaneko, Chaos **2** (3), 279–282 (1992). Lead-off article for focus issue on coupled-map lattices, models for spatially extended systems. (I)
279. ''Spatial Structure and Chaos in Insect Population Dynamics,'' M. Hassell, H. Comins, and R. May, Nature **353**, 255–258 (1991). (I)
280. ''Dynamics of a large system of coupled nonlinear oscillators,'' P. Matthews, R. Mirollo, and S. Strogatz, Physica D **52**, 293–331 (1991). (I)

3. Cellular automata

Discrete lattice systems with simple dynamical evolution rules are useful models of spatiotemporal effects. Reference 25 (Vol. II, Chap. 10) has a useful introduction.

281. ''Cellular Automata as Models of Complexity,'' S. Wolfram, Nature **341**, 419–424 (1984). (E)
282. ''Sticks and Stones: A Guide to Structurally Dynamic Cellular Automata,'' P. Halpern, Am. J. Phys. **57**, 405–408 (1989). (I)

4. Neural networks

283. ''Temporal sequences and chaos in neural nets,'' U. Riedel, R. Kuhn, and L. van Hemmen, Phys. Rev. A **38**, 1105–1108 (1988). (I)
284. ''Chaos in Random Neural Networks,'' H. Sompolinsky, A. Crisanti, and H. J. Sommers, Phys. Rev. Lett. **61**, 259–262 (1988). (A)
285. ''Chaotic Neural Networks,'' K. Aihara, T. Takabe, and M. Toyoda, Phys. Lett. A **144**, 333–340 (1990). (I)
286. ''Synchronization and Computation in a Chaotic Neural Network,'' D. Hansel and H. Sompolinsky, Phys. Rev. Lett. **68**, 718–21 (1992). (A)

5. Pattern formation in excitable media

287. ''From oscillations to excitability: A case study in spatially extended systems,'' S. C. Müller, P. Coullet, and D. Walgraef, Chaos **4** (3), 439–442 (1994). Lead-off article for focus issue on ''From oscillations to excitability—case study in spatially extended systems.'' (I)
288. ''Antichaos and Adaptation,'' S. A. Kaufman, Sci. Am. **265** (2), 78–84 (August, 1991). The emergence of order from disorder in Boolean networks and its possible implications for genetics and biological adaptation. (E)

6. Fractal growth

289. ''Diffusion-Limited Aggregation, a Kinetic Critical Phenomenon,'' T. A. Witten, Jr. and L. M. Sander, Phys. Rev. Lett. **47**, 1400–1403 (1981).(I)
290. ''Fractal Growth,'' L. M. Sander, Sci. Am. **256** (1), 94–100 (1987). General properties of systems that develop fractal geometric patterns. (E)
291. ''The Statistical Physics of Sedimentary Rock,'' P. Z. Wong, Phys. Today **41** (12), 24–32 (1988). The physics and applications of diffusion-limited aggregation, electrodeposition, and viscous fingering. (E)
292. ''Fluid Interfaces, Including Fractal Flows can be Studied in a Hele–Shaw Cell,'' J. Walker, Sci. Am. **257** (5), 134–38 (1987). How to make your own viscous fingering cell. (E)
293. ''Multifractal Phenomena in Physics and Chemistry,'' H. E. Stanley and P. Meakin, Nature **335**, 405–409 (1988).(I)
294. **Fractal Growth Phenomena** (2nd ed.), T. Vicsek (World Scientific, River Edge, NJ, 1991). (I)

Just as in crystal growth and crystal properties, defects play an important role in pattern formation.

295. ''Vortices and Defect Statistics in Two-Dimensional Optical Chaos,'' F. T. Arecchi, G. Giacomelli, P. L. Ramazza, and S. Residori, Phys. Rev. Lett. **67**, 3749–3752 (1991). (A)

7. Self-organized criticality

296. ''Self-Organized Criticality: An Explanation of $1/f$ Noise,'' P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381–384 (1987). (I)

297. ''Self-Organized Criticality,'' P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A **38**, 364–374 (1988). (I)

298. ''The Physics of Fractals,'' P. Bak and K. Chen, Physica D **38**, 5–12 (1989). An attempt to explain why fractals appear in so many physical systems. (I)

299. ''Properties of Earthquakes Generated by Fault Dynamics,'' J. M. Carlson and J. S. Langer, Phys. Rev. Lett. **62**, 2632–2635 (1989). (I)

300. ''Self-Organized Criticality,'' P. Bak and K. Chen, Sci. Am. **264** (1), 46–53 (1991). (E)

301. ''Self-Organized Criticality—An experiment with sandpiles,'' S. K. Grumbacher, K. M. Mcewen, D. A. Halverson, D. T. Jacobs, and J. Lindner, Am. J. Phys. **61** (4), 329–335 (1993). (E)

302. ''Simple models of self-organized criticality,'' G. Grinstein and C. Jayaprakash, Comput. Phys. **9** (2), 164–169 (1995). (I)

303. ''Exact Results for Spatiotemporal Correlations in a Self-Organized Critical Model of Punctuated Equilibrium,'' S. Boettcher and M. Paczuski, Phys. Rev. Lett. **76**, 348–351 (1996). (A)

304. **How Nature Works: The Science of Self-Organized Criticality**, P. Bak (Springer-Verlag, New York, 1996). (E)

G. Astrophysics

Historically, Poincaré's attempt to understand the many-body gravitational problem led to his work in nonlinear dynamics. See Ref. 8 for an excellent introduction.

305. ''Chaotic Evolution of the Solar System,'' G. J. Sussman and J. Wisdom, Science **257**, 56–62 (1992). A special computer was used to track the trajectories of the planets for the equivalent of 100 million years. The solar system has a positive Lyapunov exponent with $1/\lambda = 4$ million years. (I)

306. ''Chaos in SS Cygni?'' J. K. Cannizzo and D. A. Goodings, Astrophys. J. **334**, L31–L34 (1988). The authors analyze variable light signals from a close binary system and conclude that the signal is *not* characterized by a low-dimensional strange attractor. (A)

307. ''Chaotic Behavior in the Solar System,'' J. Wisdom, in **Dynamical Chaos**, edited by M. Berry, I. Percival, and N. Weiss (Princeton U.P., Princeton, NJ, 1987). First published in Proc. R. Soc. London, Ser. A **413**, 1–199 (1987). (I)

308. ''Pluto's Orbital Motion Looks Chaotic,'' R. A. Kerr, Science **240**, 986–987 (1988). (E)

309. ''Topological analysis of chaotic orbits—revisiting hyperion,'' P. T. Boyd, G. B. Mindlin, R. Gilmore, and H. G. R. Solari, Astrophys. J. **431** (1), 425–431 (1994). (A)

310. ''Chaos and Stochasticity in Space Plasmas,'' Geophys. Res. Lett. **18** (8) (1991). Special issue devoted to nonlinear and chaotic effects. (A)

311. ''Planets in Chaos,'' A. Youngman, New Sci. **146** (1974), 55–56 (1995). (E)

H. Controlling and synchronizing chaos

Sensitive dependence on initial conditions can be exploited to control chaotic systems.

312. ''Controlling Chaos,'' E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196–1199 (1990). (I)

313. ''Experimental Control of Chaos,'' W. L. Ditto, S. N. Rauseo, and M. L. Spano, Phys. Rev. Lett. **65**, 3211–3214 (1990). (I)

314. ''Mastering Chaos,'' W. L. Ditto and L. M. Pecora, Sci. Am. **269** (2), 78–84 (1993). (E)

315. ''Keeping chaos at bay,'' E. R. Hunt and G. Johnson, IEEE Spectr. **30** (11), 32–36 (1993). (E)

316. ''Stabilizing High-Period Orbits in a Chaotic System: The Diode Resonator,'' E. R. Hunt, Phys. Rev. Lett. **67**, 1953–1955 (1991). (I)

317. ''Controlling Chaos Using Time Delay Coordinates,'' U. Dressler and G. Nitsche, Phys. Rev. Lett. **68**, 1–4 (1992). (A)

318. ''Dynamical Control of a Chaotic Laser: Experimental Stabilization of a Globally Coupled System,'' R. Roy, T. Murphy, T. Maier, Z. Gills, and E. R. Hunt, Phys. Rev. Lett. **68**, 1259–1262 (1992). (I)

319. ''Tracking Unstable Steady States: Extending the Stability Regime of a Multimode Laser System,'' Z. Gills, C. Iwata, R. Roy, I. Schwartz, and I. Triandof, Phys. Rev. Lett. **69**, 3169–3172 (1992). (I)

320. ''Controlling Chaos in High Dimensional Systems,'' D. Auerbach, C. Grebogi, E. Ott, and J. A. Yorke, Phys. Rev. Lett. **69**, 3479–3482 (1992). (A)

321. ''Controlling Cardiac Chaos,'' A. Garfinkel, M. L. Spano, W. L. Ditto, and J. N. Weiss, Science **257**, 1230–1235 (1992). Rabbit heart arrhythmias controlled by nonlinear dynamics techniques. (I)

322. ''Using small perturbations to control chaos,'' T. Shinbrot, C. Grebogi, E. Ott, and J. A. Yorke, Nature **363**, 411–417 (1993). (E)

323. ''Control of the chaotic pendulum,'' G. L. Baker, Am. J. Phys. **63** (9), 832–838 (1995). Detailed numerical computations illustrating the control of chaos. (I)

324. ''Controlling Chaos,'' Y. C. Lai, Comput. Phys. **8** (1), 62–67 (1994). (I)

325. ''Controlling Chaos,'' E. Ott and M. Spano, Phys. Today **48** (5), 34–40 (1995). (E)

326. ''Progress in the control of chaos,'' T. Shinbrot, Adv. Phys. **44**, 73–111 (1995). (I)

327. **Controlling Chaos**, T. Kapitaniak (Academic, San Diego, 1996). A good introduction to various control schemes. Includes 13 reprints of seminal papers in the field. (I)

Synchronizing chaotic systems

328. ''Synchronization in chaotic systems,'' L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **64**, 821–824 (1990). Chaotic systems can be synchronized with suitable interconnections. (I)

329. ''Driving Systems with Chaotic Signals,'' L. Pecora and T. Carroll, Phys. Rev. A **44**, 2374–2383 (1991). (I)

330. ''Circuit Implementation of Synchronized Chaos with Applications to Communications,'' K. M. Cuomo and A. V. Oppenheim, Phys. Rev. Lett. **71** (1), 65–68 (1993). How synchronized chaotic systems can be used to encrypt and decode messages. (I)

331. ''Synchronization of chaotic systems—the effects of additive noise and drift in the dynamics of the driving,'' R. Brown, N. F. Rulkov, and N. B. Tufillaro, Phys. Rev. E **50** (6), 4488–4508 (1994). (I)

332. ''The effects of additive noise and drift in the dynamics of the driving on chaotic synchronization,'' R. Brown, N. Rulkov, and N. B. Tufillaro, Phys. Lett. A **162**, 201–205 (1994). (I)

333. ''A simple circuit for demonstrating regular and synchronized chaos,'' T. L. Carroll, Am. J. Phys. **63** (4), 377–379 (1995). (I)

334. ''Ordering chaos with disorder,'' S. H. Strogatz, Nature **378**, 444 (1995). How a bit of disorder can help synchronize an array of oscillators. (E)

335. ''Taming spatiotemporal chaos with disorder,'' Y. Braiman, J. F. Lindner, and W. L. Ditto, Nature **378**, 465–467 (1995). (I)

I. Fluid dynamics and fluid mixing

Fluid motion is a rich source of nonlinear behavior. The best introductory references are Ref. 26 and

336. **Hydrodynamic Instabilities and the Transition to Turbulence** (2nd ed.), edited by H. L. Swinney and J. P. Gollub (Springer-Verlag, New York, 1985). (I)

1. Fluid mixing

337. ''The Mixing of Fluids,'' J. M. Ottino, Sci. Am. **260** (1), 56–67 (1989). Illustrates the connection between horseshoe dynamics and fluid tracer experiments. Beautiful examples of stretching, compression, and folding. (E)

338. ''Morphological Structure Produced by Mixing in Chaotic Flows,'' J. M. Ottino, C. Leong, H. Rising, and P. Swanson, Nature **333**, 419–425 (1988). (E)

2. Turbulence

339. ''On the Problem of Turbulence.'' L. D. Landau, Akad. Nauk. Doklady **44**, 339 (1944). English translation reprinted in Ref. 31, Vol. I. A proposal for a cascade of an infinite number of frequencies to explain turbulence. (I)

340. ''Turbulence: Challenges for Theory and Experiment,'' U. Frisch and S. A. Orszag, Phys. Today **43** (1), 24–32 (January, 1990). (E)

3. Leaky faucet

341. ''The Chaotic Behavior of the Leaky Faucet,'' P. Martien, S. C. Pope, P. L. Scott, and R. S. Shaw, Phys. Lett. A **110**, 399–404 (1985). (I)

342. ''Chaotic Rhythms of a Dripping Faucet,'' R. F. Cahalan, H. Leidecker, and G. D. Cahalan, Comput. Phys. **4**, 368–383 (1990). Using a computer to take data from a dripping faucet. (I)

343. ''The Route to Chaos in a Dripping Faucet,'' K. Dreyer and F. R. Hickey, Am. J. Phys. **59**, 619–627 (1991). (I)

4. Rayleigh–Bénard (convection) experiments

344. ''Fixed Winding Number and the Quasiperiodic Route to Chaos in a Convective Fluid,'' J. Stavans, F. Heslot, and A. Libchaber, Phys. Rev. Lett. **55**, 596–599 (1985). (A)

345. ''Global Universality at the Onset of Chaos: Results of a Forced Rayleigh–Bénard Experiment,'' M. H. Jensen, L. Kadanoff, A. Libchaber, I. Proccacia, and J. Stavans, Phys. Rev. Lett. **55**, 2798–2801 (1985). (I)

346. ''Simple Multifractal Cascade Model for Fully Developed Turbulence,'' C. Meneveau and K. R. Sreenivasan, Phys. Rev. Lett. **59**, 1424–1427 (1987). (A)

5. Surface waves

347. ''Order-disorder transition in capillary ripples,'' N. B. Tufillaro, R. Ramshankar, and J. P. Gollub, Phys. Rev. Lett. **62** (4), 422–425 (1989). (I)

348. ''Patterns and Quasi-Patterns in the Faraday Experiment,'' W. S. Edwards and S. Fauve, J. Fluid Mech. **278**, 123–148 (1994). (I)

349. ''On Faraday Waves,'' J. Miles, J. Fluid Dyn. **248**, 671–683 (1993). (I)

J. Quantum chaos

How does a quantum system behave when its classical counterpart exhibits chaos? See Ref. 74 for a good introduction.

350. ''Quantum Chaos,'' M. C. Gutzwiller, Sci. Am. **266** (1), 78–84 (1992). An introductory essay on the questions raised by quantum chaos. (E)

351. ''Quantum Chaos, Is There Any?'' J. Ford, in Ref. 31, Vol. II, pp. 128–147. (I)

352. ''The Quantum Chaos Problem,'' P. V. Elyatin, Sov. Phys. Usp. **31**, 597–622 (1988). (A)

353. **Chaos in Classical and Quantum Mechanics**, M. C. Gutzwiller (Springer-Verlag, New York, 1990). (A)

354. **The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations**, L. Reichl (Springer-Verlag, New York, 1992). (A)

355. ''Does Quantum Mechanics Obey the Correspondence Principle? Is it Complete?'' J. Ford and G. Mantica, Am. J. Phys. **60**, 1086–1098 (1992). A provocative paper arguing that quantum mechanics is incomplete because it does not describe chaotic behavior. (I)

1. Semiclassical methods

356. ''Postmodern Quantum Mechanics,'' E. J. Heller and S. Tomsovic, Phys. Today **46** (7), 38–46 (1993). (E)

357. ''Periodic Orbit Theory in Classical and Quantum Mechanics,'' P. Cvitanovic, Chaos **2** (1), 1–4 (1992). This issue focuses on periodic orbit theory in semiclassical quantum mechanics. (A)

2. Map models in quantum mechanics

358. ''Quantum-Classical Correspondence in Many-Dimensional Quantum Chaos,'' S. Adachi, M. Toda, and K. Ikeda, Phys. Rev. Lett. **61**, 659–661 (1988). Coupled kicked-rotors show effects closer to those of classical chaos. (I)

359. ''Experimental Realizations of Kicked Quantum Chaotic Systems,'' R. E. Prange and S. Fishman, Phys. Rev. Lett. **63**, 704–707 (1989). An experiment (modes in optical fibers) described by the kicked rotator model. (I)

3. Energy level statistics

360. ''Quantum Manifestations of Classical Chaos: Statistics of Spectra,'' J. V. José, in Ref. 31, Vol. II, pp. 148–193. A good review of the issues concerning energy level statistics as a symptom of classical chaos in quantum mechanics. (A)

Atomic physics provides an ideal testing grounds for the notions of quantum chaos because calculations can be done in many cases to high accuracy and high precision experiments on carefully controlled systems are its forte. The following two items provide very readable general reviews:

361. ''Chaos in Atomic Physics,'' R. V. Jensen, in **Atomic Physics** (Vol. 10), edited by H. Narumi and I. Shimamura (Elsevier, Amsterdam, 1987), pp. 319–332. (I)

362. ''The Bohr Atom Revisited: A Test Case for Quantum Chaos,'' R. V. Jensen, Comments At. Mol. Phys. **25**, 119–31 (1990). (I)

363. ''Microwave Ionization of Hydrogen Atoms: Experiment versus Classical Dynamics,'' K. A. H. van Leeuwen *et al.*, Phys. Rev. Lett. **55**, 2231–2234 (1985). Highly excited hydrogen atoms ionized by microwave oscillating fields have been a test bed for both theory and experiment dealing with quantum chaos. (I)

364. ''Microwave Ionization of H Atoms: Breakdown of Classical Dynamics for High Frequencies,'' E. J. Galvex, B. E. Sauer, L. Moorman, P. M. Koch, and D. Richards, Phys. Rev. Lett. **61**, 2011–2014 (1988). (I)

365. ''Ionization steps and phase-space metamorphoses in the pulsed microwave ionization of highly excited hydrogen atoms,'' J. E. Bayfield, S. Y. Lurie, L. C. Perotti, and M. P. Skrzypskowski, Phys. Rev. A **53**, R12–15 (1996). (I)

366. ''Irregular Atomic Systems and Quantum Chaos,'' Comments At. Mol. Phys. **25** (1–6), 1–362 (1991). An entire volume dedicated to the question of atomic physics and quantum chaos. (I,A)

367. ''Chaos in Atomic Physics,'' T. S. Monterio, Contemp. Phys. **35** (5), 311–327 (1994). (E)

368. ''Sodium atoms kicked by standing waves provide new probe of quantum chaos,'' G. P. Collins, Phys. Today **48** (6), 18–21 (1995). (I)

Single-electron atoms in strong magnetic fields have provided an impressive test of quantum predictions in a regime for which the corresponding classical system is nonintegrable and shows chaotic behavior. The quantum-mechanical calculations are described in the following papers:

369. ''Effect of Closed Classical Orbits on Quantum Spectra: Ionization of Atoms in a Magnetic Field,'' M. L. Du and J. B. Delos, Phys. Rev. Lett. **58**, 1731–1733 (1987). (A)

370. ''Positive-Energy Spectrum of the Hydrogen Atom in a Magnetic Field,'' D. Delande, A. Bommier, and J. C. Gay, Phys. Rev. Lett. **66**, 141–144 (1991). (A)

371. ''Diamagnetic Rydberg atom: Confrontation of calculated and observed,'' C.-H. Iu, G. R. Welch, M. M. Kasch, D. Kleppner, D. Delande, and J. C. Gay, Phys. Rev. Lett. **66**, 145–148 (1991). The corresponding experiment. (I)

4. Other quantum systems

372. ''Aspects of Chaos in Nuclear Physics,'' O. Bohigas and H. A. Weidenmüller, Annu. Rev. Nucl. Part. Sci. **38**, 421–453 (1988). (I,A)

373. ''Nature of Quantum Chaos in Spin Systems,'' G. Müller, Phys. Rev. A **34**, 3345–3355 (1986). (A)

K. Nonlinear forecasting

Forecasting with nonlinear dynamics and chaos. Sensitive dependence on initial conditions would seem to make prediction of chaotic behavior an impossibility. But you can exploit some aspects of nonlinear dynamics to do better than you might think.

374. ''Predicting Chaotic Time Series,'' J. D. Farmer and J. J. Sidorowich, Phys. Rev. Lett. **59**, 845–848 (1987). (I)

375. ''Nonlinear Predictions of Chaotic Time Series,'' M. Casdagli, Physica D **35**, 335–356 (1989). (I)

376. **Nonlinear Modeling and Forecasting**, edited by M. Casdagli and S. Eubank, Proceedings Volume XII, Santa Fe Institute Studies in the Sciences of Complexity (Addison-Wesley, Reading, MA, 1992). Proceedings of the Workshop on Nonlinear Modeling and Forecasting, September, 1990, Santa Fe, New Mexico. (I)

377. ''The future of time series: Learning and understanding,'' N. Gershenfeld and A. S. Weigend, in Ref. 39. (I)

L. Complexity

The general issues of complex systems span many disciplines and have led to some speculations about general theories of complexity. See also, J. Ford in Ref. 31, Vol. II, pp. 139–147.

1. Algorithmic complexity and information theory

378. ''Randomness and Mathematical Proof,'' G. J. Chaitin, Sci. Am. **232** (5), 47–52 (1975). (E)

379. ''How Random is a Coin Toss,'' J. Ford, Phys. Today **36** (4), 40–47 (1983). (E)

380. **Algorithmic Information Theory**, G. J. Chaitin, (Cambridge U.P., Cambridge, 1987). (A)

2. Complexity theory

381. **Chance and Chaos**, D. Ruelle (Princeton U.P., Princeton, NJ, 1991). A good discussion of what ''complexity theory'' might be all about. (E)

382. **Complexity: The Emerging Science at the Edge of Order and Chaos**, M. M. Waldrop (Simon and Schuster, New York, 1992). (E)

383. **Frontiers of Complexity: The Search for Order in a Chaotic World**, P. Coveney and R. Highfield (Fawcett/Columbine, New York, 1995). (E)

384. **Hidden Order: How Adaptation Builds Complexity**, J. H. Holland (Addison-Wesley, Reading, MA, 1995). A collection of essays by one of the complexity pioneers. (E)

385. **At Home in the Universe, The Search for the Laws of Self-Organization and Complexity**, S. Kaufmann (Oxford U.P., New York, 1995). (I)

386. ''From Complexity to Perplexity,'' J. Horgan, Sci. Am. **272** (6), 104–109 (1995). A skeptical review of what complexity theory is up to. (E)

M. Economics geophysics, ecology, computer networks, literary theory, and music

387. ''Is the Business Cycle Characterized by Deterministic Chaos?'' W. A. Brooks and C. L. Sayers, J. Monetary Econ. **22**, 71–90 (1988). (I)

388. ''Chaos: Significance, Mechanism, and Economic Applications,'' J. Baumol and J. Benhabib, J. Econ. Perspectives **3**, 77–105 (1989). (I)

389. ''Nonlinearities in Economic Dynamics,'' J. A. Scheinkman, Econ. J. **100**, 33–48 (1990). (I)

390. ''Chaos and nonlinear forcastability in economics and finance,'' B. Lebaron, Philos. Trans. R. Soc. London, Ser. A **348** (1688), 397–404 (1994). (I)

391. **Nonlinear Dynamics, Chaos and Econometrics**, edited by M. Hashem Pesaran and S. M. Potter (Wiley, New York, 1993). (I)

392. ''Chaos, Fractals, Nonlinear Phenomena in Earth Sciences,'' D. L. Turcotte, Rev. Geophys. **33** (Suppl. A), 341–344 (1995). (I)

393. ''Nonlinear Dynamics and Predictability,'' W. I. Newman, A. Gabielov, D. L. Turcotte, and C. Yong, Pur. Appl. Geophys. Phen. **145** (2), 371–380 (1995). (I)

394. ''Chaos in geophysical fluids 1. General Introduction,'' R. Hide, Philos. Trans. R. Soc. London, Ser. A **348** (1688), 431–443 (1994). (I)

395. ''El-Nino on the devils staircase-annual subharmonic steps to chaos,'' F. F. Jin, J. D. Neelin, and M. Ghil, Science **264** (5155), 70–72 (1994). (E)

396. ''El-Nino Chaos-Overlapping of resonances between the seasonal cycle and the pacific ocean-atmosphere oscillator,'' E. Tziperman, L. Stone, M. A. Cane, and H. Jarosph, Science **264** (5155), 72–74 (1994). (E)

397. ''An Ecology of Machines, How Chaos Arises in Computer Networks,'' B. A. Huberman, The Sciences (New York Academy of Sciences), 38–44 (July/August, 1989). (I)

398. ''Ecologists Flirt with Chaos,'' R. Pool, Science **243**, 310–313 (1989). (E)

399. **Chaos and Order: Complex Dynamics in Literature and Science**, edited by N. K. Hayles (University of Chicago Press, Chicago, 1991). (E)

400. ''Symphony in Chaos,'' E. Kostelich, New Sci. **146** (1972), 36–39 (1995). Chaos ideas applied to the composition of music. (E)

VII. SOFTWARE

401. **Strange Attractors**, J. C. Sprott (M&T Books, New York, 1993). (I)

402. **Chaos Data Analyzer**, J. C. Sprott (Physics Academic Software, North Carolina State University, Box 8202, Raleigh, NC 27695). (I)

403. **Chaos Demonstrations**, J. C. Sprott (Physics Academic Software, North Carolina State University, Box 8202, Raleigh, NC 27695). (E)

404. **Chaotic Dynamics Workbench**, R. Rollings (Physics Academic Software, North Carolina State University, Box 8202, Raleigh, NC 27695). (I)

405. **Chaos: The Software**, R. Rucker and J. Gleick (Autodesk Inc., Sausalito, CA, 1990). (E)

406. **Chaotic Mapper**, J. B. Harold (Physics Academic Software, North Carolina State University, Box 8202, Raleigh, NC 27695). (I)

407. **Fractint**, T. Wegner and M. Peterson (freeware available on Compuserve, BIX, and other bulletin boards). (E)

408. **Chaos: A Program Collection for the PC**, H. J. Korsch and H.-J. Jodl (Springer-Verlag, Berlin, 1994). (I)

409. **Dynamics: Numerical Explorations**, H. E. Nusse and J. A. Yorke (Springer-Verlag, New York, 1994). (I)

VIII. INTERNET RESOURCES

410. A good jumping-off point in cyberspace for nonlinear dynamics is the (hypertext) version of the nonlinear FAQ maintained by J. Meiss and S. Doole (http://amath.colorado.edu/appm/faculty/jdm/faq.html). (E)

All of the major centers of nonlinear science research have extensive Web sites. A few bookmarks to get you started include:

411. Chaos at the University of Maryland (http://www-chaos.umd.edu). In particular see their ''Chaos Database,'' which has a nice search engine for an extensive bibliographic database (http://www-chaos.umd.edu/publications/searchbib.html). (E)

412. The Institute of Nonlinear Science at the University of California at San Diego (http://inls.ucsd.edu). (E)

413. The Center for Nonlinear Dynamics at University of Texas at Austin (http://chaos.ph.utexas.edu). (E)

414. The Center for Nonlinear Science at Los Alamos National Lab (http://cnls.lanl.gov). In particular, see the CNLS Nonlinear Science *e*-print archive (http://cnls.lanl.gov/pbb.announce.html), and the Nonlinear Dynamics Archive (http://cnls.lanl.gov/nbt/intro.html). (E)

415. G. Chen, ''Control and synchronization of chaotic systems (a bibliography)'' is available by anonymous ftp from (uhoop.egr.uh.edu/pub/TeX/chaos.tex). (I)

416. ''Nonlinear Dynamics Bibliography'' maintained by the University of Mainz (http:/www.uni-mainz.de/FB/Physik/Chaos/chaosbib.html). (E)

417. The Applied Chaos Laboratory at the Georgia Institute of Technology (http://acl2.physics.gatech.edu/). (E)