25 September 1986

BOUNCING BALL EXPERIMENT

or, Listening to Chaos

I. Introduction

Imagine dropping a ball on top of a table which is oscillating in a vertical direction with a frequency ω and amplitude A.

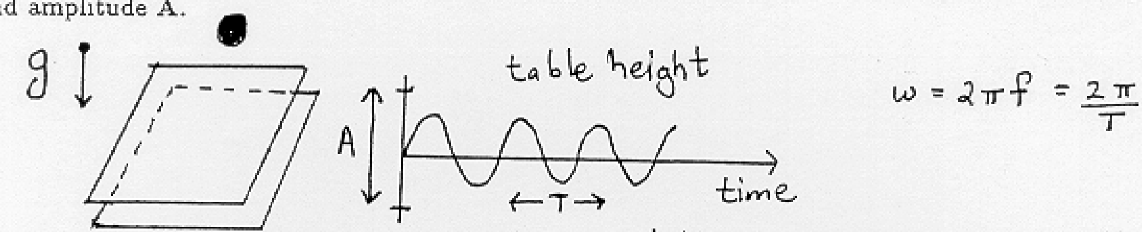

Fig 1: A ball is free to bounce on a table that moves up and down.

In this experiment we want to study how varying the amplitude A of the table effects the ball's dynamics. When the ball hits the table a little energy is lost — the collision is *inelastic*. So we know right away that if the table does not vibrate (A = 0) then the ball will eventually come to rest. Also, as you might suspect, if A is small enough then the ball will move with the table and not bounce. However, for a big enough amplitude, the ball will separate from the table and begin to bounce. As we shall see, the ball will initially bounce periodically until we reach a critical value of the amplitude A_c at which point the ball will appear to bounce in a chaotic manner.

Problem 1:

Assume the table moves up and down according to,

$$Table(t) = A\cos(\omega t)$$

at what value of A do you expect the table and ball to initially separate as you slowly increase A? (Hint: calculate the table's acceleration.)

One way to visualize the ball's motion is to graph both the ball's height and the table's height on the same plot as shown in Figure 2.

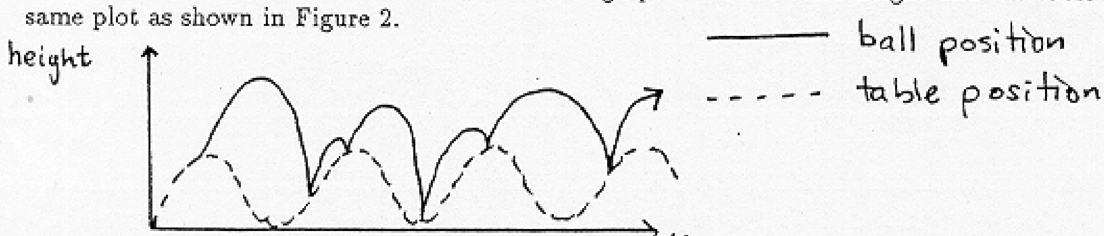

Fig 2: Ball & Table position

The balls motion in Figure 2 is not periodic. The simplest periodic motion we can imagine would look like:

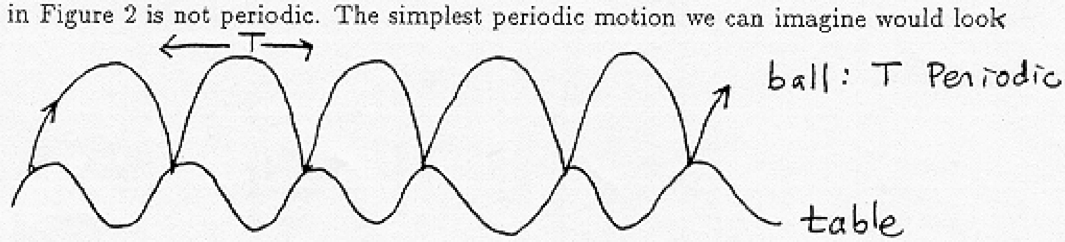

Fig 3: Periodic motion of period T.

As shown in Figure 3, the table moves with a period T and the ball's motion also repeats itself in a time T. The next simplest periodic motion would have a period 2T.

The ball might bounce high, then low, then high again. The pattern will repeat itself at *twice* the forcing frequency. As we shall see, the orbit of period 2T is born from an an orbit of period T when we increase the table amplitude A. Under these circumstances we say that the period T orbit **period doubles** to a period 2T orbit.

Studying the *Logistic map*

$$z_{n+1} = 4Az_n(1 - z_n)$$

we discovered that chaotic solutions were preceded by a sequence of period doubled solutions. Reasoning by analogy, we might guess that in the bouncing ball experiment, as we increase the table amplitude (which is analogous to the control parameter A in the logistic map) we will see a sequence of periodic orbits of length T, 2T, 4T, ... afterwhich the ball moves chaotically. Notice that if our analogy is correct we will not see orbits of length 3T, 5T, 6T, 7T, ... in this sequence. Moreover, in the logistic map experiment, it was mentioned that the *Feigenbaum number*

$$\delta_F = \lim_{n\to\infty} \frac{A_n - A_{n-1}}{A_{n+1} - A_n} \approx 4.7$$

was *universal*, i.e., it should occur in almost any system exhibiting period doubling. The point of this lab then is to observe the *period doubling route to chaos* in a simple experimental system and to see how a measurement of Feigenbaum's delta arises from an experimental data.

II. Experimental Set-up:

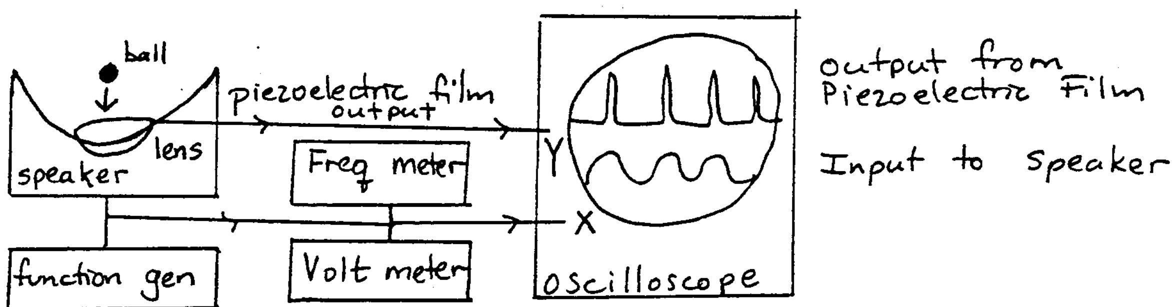

Fig 5: Bouncing Ball apparatus.

A schematic of a *bouncing ball machine* is shown in Figure 5. A speaker driven by a function generator serves as our *vibrating table*. A steel ball bounces against a lens fastened the the speaker. Every time the steel ball hits the lens a sharp click is heard. The impact between the ball an the table is detected with a *piezoelectric* sensor, a thin piece of pressure sensitive material attached to the lens. Piezoelectricity or *pressure electricity* is a capability of certain crystalline materials to change their dimensions when subjected to an electrical field, or conversely, to produce electrical signals when mechanically deformed. The piezoelectric phenomenon was first discovered to occur in natural quartz crystals by Pierre and Jacques Curie in the 1880s. The Curie brothers also predicted pyroelectricity, an electrical charge developed from a thermal change. Practical use of the piezoelectric property of quartz was made in 1916 by Langevin, who develped an ultrasonic sending and receiving system. A piezoelectric quartz crystal was set into oscillation by an electrical signal, and the high frequency mechanical vibration was transmitted through water to a reflecting body. A second quartz crystal received the reflected vibratory energy (ultrasound) and, from the time lapse between sending and receiving, the distance from the source to the reflecting body could be calculated. This major application of piezoelectricity was the forerunner of modern day sonar. More details about the techanical aspects of the Piezo Film used in this experiment are found in the **Kynar Piezo Film Technical Manual.**

III. Procedure:

A) Examine the set-up and make required connections (see Figure 5). Turn-on equipment. On the function generator there is a small knob controlling the output voltage to the speaker. Set the function generator to 60 Hz sine wave. Slowly increase the voltage until the ball begins to bounce, then decrease the output voltage (the volume) until you can *hear* the ball bouncing in a periodic manner.

Problem 2:

Examine the speakers amplitude by viewing the driving signal on the osciloscope for different driving frequencies between 40 and 100 Hz. Is the amplitude a function of the frequency. If so, can you quickly explain why? (Hint: How does a speaker convert electrical energy to mechanical energy) Record where the *resonances* occur at.

B) Practice *listening* to the clicking of the ball until you can distinguish *different periodic motions* and chaotic motions. When you think you have it, confirm you expectation by looking at the waveform on the oscilloscope. Try to trigger off both the input signal and the signal from the piezoelectric film to see what works best. Try the X-Y mode. Adjust triger and voltage levels to optimize the signals.

C) Set the function generator to a fixed operating frequency. Attempt to make a *bifurcation* diagram as follows. Place the function generator at a high voltage so that the ball bonnces in a chaotic manner then decrease the amplitude noting for what voltages (read from the voltmeter) periodic and chaotic motions appear and dissappear. Note if the bifurcation points shift if you are increasing or decreasing the table amplitude. This effect is an example of *hysteresis*. Fill in the details as best you can to Figure 6.

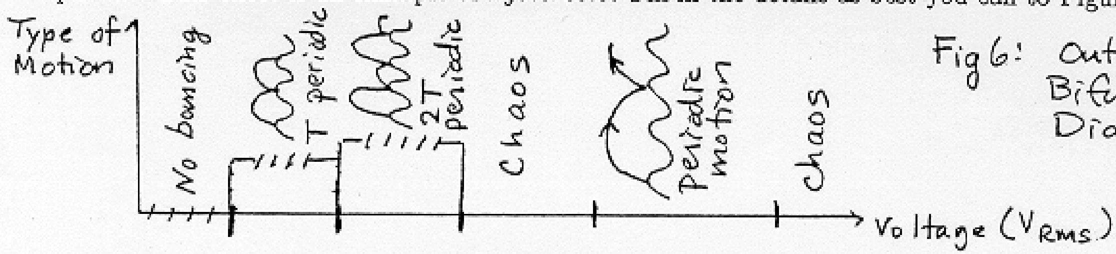

Fig 6: Outline of Bifurcation Diagram

D) Attempt to calculate the Feigenbaum delta as follows. In the period doubling regieme, note the voltage at which the ball stops bouncing as you *decrease* the table amplitude, call this A_0. Next measure as best you can when the period T orbit period doubles to period 2T. Let A_1 be the voltage where this occurs as you decrease the driving voltage to go from the period 2T orbit to the period T orbit. Lastly, try to see if you can find the bifurcation point to the period 4T orbit, this may take some patience, you can probably hear it, but you will have to play with the oscilloscope to get it to trigger properly, call this value A_2. (If you can't get the period 4T orbit, then let A_2 be the value at which chaotic motion appears.) Now calculate

$$\delta_f \approx \frac{A_1 - A_0}{A_2 - A_1}$$

How does it compare with the logistic map value?

Questions:

1. In the *logistic map* we saw orbits of length 2T, 4T, 8T, ... etc. However, in the experiment, the longest orbit we saw before chaos was 4T. This is called a *truncation of the cascade*. Why might the period doubling have only a finite cascade for an experimental system? That is, why don't we see very long periodic orbits right before chaos?

2. A human hair is approximately 10 microns (1 micron = 10^{-6} meters). Assume the ball bounces in a periodic pattern as shown in Figure 3. From elementary considerations ($x(t) = x_0 + v_0 t - \frac{1}{2}gt^2$) estimate (in human hairs) the maximum seperation between the ball and the lens. To compare your estimate with the experiment, what additional measurements do you need to make?