Measurement driven models of nonlinear electronic components.

Nicholas Tufillaro, Daniel Usikov, and Lee Barford
Agilent Techologies, Inc.
1501 Page Mill Road, MS 4A-D
Palo Alto, CA 94304 USA

David M. Walker
Center for Applied Dynamics and Optimization
The University of Western Australia
Nedlands, Perth, WA 6907, Australia

Dominique Schreurs
K. U. Leuven
Div. ESAT-TELEMIC
Kard. Mercierlaan 94
B-3001 Heverlee, Belgium
(February 29, 2000)

A modeling method that allows one to rapidly build data driven models for nonlinear components is discussed. The models are constructed from input/output time domain data and their "embeddings". The notion of models built from embeded data is described in the Taken's Embedding Theorem and has been extensively explored for modeling autonomous systems in the physics liteature. The authors extend these results to nonautonomous systems by creating tools that allow engineers to rapidly build models for driven nonlinear components. These models can be used in simulation, process control, diagonostics, and sensor calibration.

We present the results of applying nonlinear modelling techniques to the modelling of measurements taken from a Nonlinear Network Measurement Systems (NNMS). A comparison is made between reconstructed polynomial models and radial basis models.

I. INTRODUCTION

This paper describes a dynamical systems approach to nonlinear system identification. A nonlinear system or component typically is not well described by a (linear theory) transfer function [1] so a new approach is needed for building data driven models for nonlinear systems. Some methods that have been developed include Volterra Series, neural nets, and cluster weighted models to name a few [2].

A dynamical systems approach to "black box" or "behavioral modeling" was first suggested by Casdagli [3]. Using time domain input/output or scattering data an attempt is made to embed the orginial data in a higher dimensional space, built from transforms of the original data of sufficient dimension so that the determinism of the dynamical system is recovered. These methods have been developed for data in the time domain and are sufficiently different from the typical linear system frequency domain techniques that we begin with some background material before reporting results with experimental data.

This approach to nonlinear system identification is sometimes called "Dynamic Reconstruction Theory [4]" and begins with a return to the state space representation

$$\dot{x} = f(x(t), u(t)) \qquad y(t) = h(x(t)) \tag{1}$$

or their numerical version of difference equations,

$$x_{n+1} = f(x_n, u_n) \tag{2}$$

In these equations f, x, u are typically vectors and $u(t)$ is the input, drive, or stimulus, $x(t)$ is the state, and $h(t)$ is a measurement function. Attempts to build data state space models appear hard on at least two counts: first, without any specific form for a model the relevant dynamical variables x appears to be unknown, and second, even if we know what variables are needed to be included, they still may not be accessible to experimental measurements. Both of these issues, essentially the nonlinear order, or dimension of the model, and model selection and calibration are discussed below.

A simple approach to nonlinear modeling in the time domain could begin by plotting the input and output of on a graph. Next we could create a function from the stimulus $u(t)$ to the the response $y(t)$,

$$y(t) = F[u(t)] \tag{3}$$

but this function might not be unique. The key idea of dynamic reconstruction is to embed these variables to resolve this indeterminancy by building a function not just with $y(t)$, but also transforms of y, for example its numerical derivatives. An "embedding" is a map that places an "m" dimensional manifold, in this case a one-dimensional curve, in an "n" dimensional space. A polynomical model then would be of the form

$$y(t) = a_0 + a_1 u + a_2 \dot{u} + a_3 u\dot{u} + a_4 u^2 + a_5 \dot{u}^2 \dots \quad (4)$$

the unknown coefficients $(a_0, a_1, a_2, ...)$ can be determined by least squares. Ploting the embedded trajectory in the enlarged phase space can untangle and remove the indeterminacy. This idea can also be applied to difference equations,

$$f(n+1) = F[f(n)] \quad (5)$$

and in effect create a numerical approximation for the differential equations generating the flow.

Due to a theorem of Takens (with an extension to the driven case by Stark [5]) these embeded models are diffeomorphic to the dynamics of the original system. This means that there is a continuous and differentiable map from the original system trajectory (say generated by my best first principles model for the system) to the new embeded system created from the measured variables of sufficient dimension. In particular, deterministic prediction is possible from an embeded model which will mimic the actual dynamics.

Thus, embedding opens the way toward a general solution of extracting black box models for the observable dynamics of nonlinear systems directly from input/output time series data. It can solve the fundamental existence problem, however, the gulf between these theoretical results and practical implementation is wide.

Practically, the components behavior is described by embedding both the inputs and outputs in the form

$$\begin{aligned} z(t) = G[&y(t-\tau), y(t-2\tau), ..., y(t-l\tau), \\ &u(t), u(t-\tau), ..., u(t-(k-1)\tau)] \end{aligned} \quad (6)$$

where G is fitted to the data using nonlinear modeling methods such as global polynomials, neural nets, or radial basis functions [2]. The form of the equations shows a "lag" embedding with a time delay τ, input lag dimension k and output lag dimension l, though in practice we find that better quality models can often be built using other embeddings such as linear transforms, integral and differential transforms, and wavelets to help bring out the salient dynamical features in the data.

Given the above model form, the problem now reduces to a number of technical issues such as: Determination of the dimensions (k and l), determination of lags (τ) or other forms of embeddings and embedding parameters, determination of model class G, and fitting the model parameters, model validation, and design of excitation signals (where possible) for a given model/signal class.

It might be helpful to point out that this relation between a continuous dynamical system and an embedded model built from time-delayed input/output signals can be made explicit in the case of linear systems. The details for an algorithm are described in a book by Franklin [6], which shows how to go from the linear system with matrices A, B, and C to a model based only on delayed variables. Unfortunately, no explicit constructive proof exists for nonlinear systems.

For embeddings built from a time delay lag τ,

$$y(t) = G[y(t-\tau), ..., y(t-l\tau), u(t), ..., u(t-(k-1)\tau)] \quad (7)$$

we can use an extension of the algorithm for the theory of embedded autonomous systems known as "False Nearest Neighbors [7]". Basically, we find the smallest "k" and "l" by creating a statistic that checks if vectors close in a delay space are also close in a delay space of greater dimension [8]. If they are not, then we have false neighbors and G is not single valued. This diagnostic is independent of G.

For models built from time delays we need to estimate τ. Again, we find that a diagnostic from autonomous systems theory sufficies. We use either the mutual information or the first zero of the autocorrelation function in determining τ [7]. In cases where there is a single dominant frequency band, both of these diagnostics often turn out to be about one-quarter of the dominant frequency, or in other words, τ is chosen so that the delay variables are decorelated as much as possible.

In practice, though, we find that these diagnostics are not nearly so useful as software tools that allow us to rapidly build and test models with different combinations of embedding functions and parameters.

Once we have a suitable embedding, we next turn to function approximation of G. We try to keep things here as simple as possible. First we usually try a global polynomial fitted by least squares. For circuit applications we have had some success with radial basis functions

$$y = \alpha + \beta u + \sum_{i=1}^{N} \omega_i \phi(||c_i - u||) \quad (8)$$

that use optimization algorthims which can automatically determine the number and placement of the basis functions [9,13].

Again, with good software tools we can rapidly test out different basis functions and model structures. Finally, for validation, we usually just do out-of-sample testing with some simple metrics for the average and maximum errors.

II. NNMS RESULTS.

We develop a nonlinear black box behavioural model of a microwave amplifier by reconstructing the model using measurement data from a NNMS. Details of the test setup are found elsewhere [10]. The test apparatus provides band limited frequency domain output which we convert to the time domain for the analysis. Time domain outputs and the stimulus usually consists of multiple periodic signals.

The goal is to produce a model which accurately predicts the currents given past and present values for the voltages. The device is a lattice-matched InP HEMT with 0.2 μm gate length and 100 μm gate width, fabricated at IMEC, Belgium.

The data consists of 30 sets of NNMS measurements. These data sets are split into two groups of 15 measurements at different DC bias points. The 15 data sets are further split by a parameter indicating the phase difference between the two incident voltage waves. There are 256 data points in each of the 30 data sets.

The aim is to reconstruct models using some (or all) of the data with the property that the models test well on the other data sets. Initially we will attempt to reconstruct a global model using all of the data (actually only 15 of the data sets). Then we attempt to construct accurate models using only a subset of all the data available.

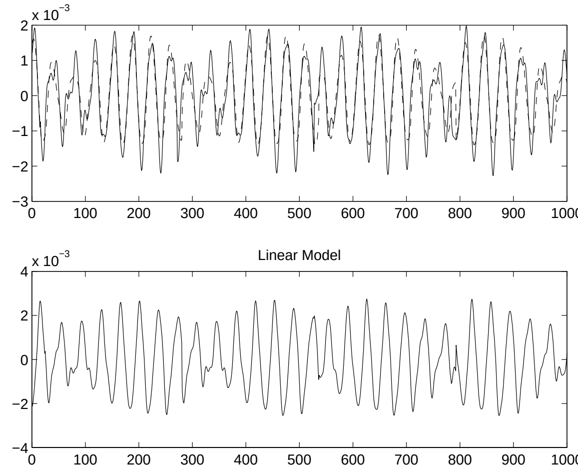

FIG. 1. The fit and errors of a linear model for the gate current 1.

III. POLYNOMIAL MODELS

In this section we will reconstruct polynomial models. We assume the data has been embedded in an appropriate dimension with a suitiable lag. As mentioned in the introduction there are methods of finding a good dimension [11,12,8]. The lag is chosen by examining the autocorrelation function applied to the input data. We found that a lag of 9 and an embedding dimension of 2 for each input sequence was suitable.

A. Modelling using all data sets

We reconstruct a polynomial model using all the data sets at one bias point. The phase difference parameter and the bias point is ignored, and so the inputs are just the voltages at ports 1 and 2 with their corresponding lagged value. The outputs are the currents at ports 1 and 2.

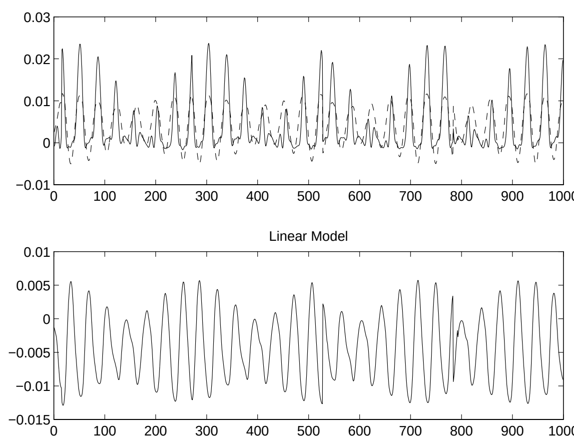

FIG. 2. The fit and errors of a linear model for the drain current 2.

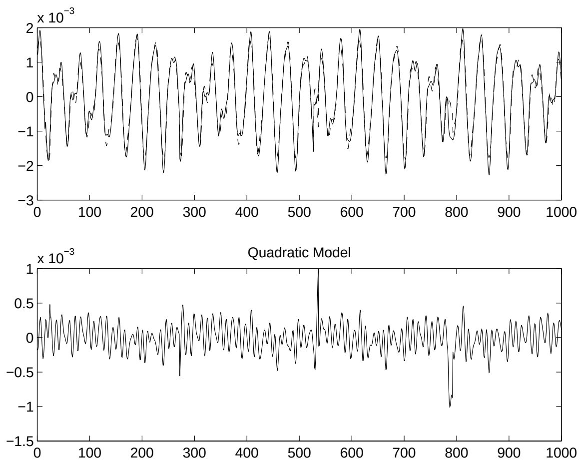

FIG. 3. The fit and errors of a quadratic model for current 1.

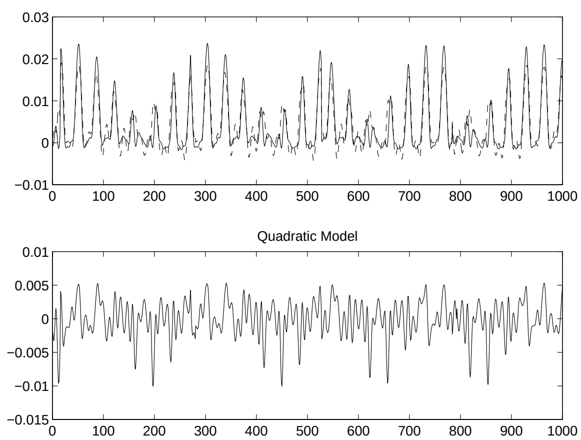

FIG. 4. The fit and errors of a quadratic model for current 2.

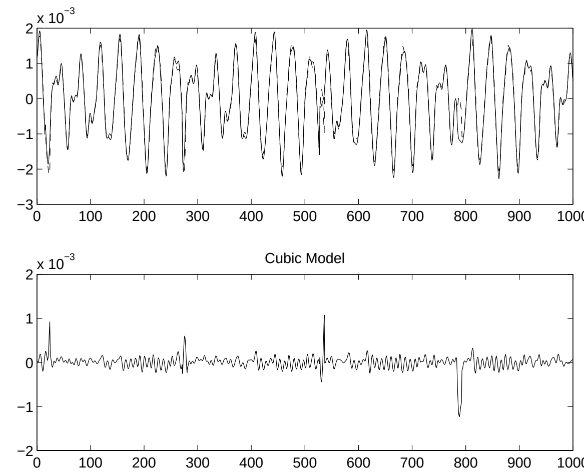

FIG. 5. The fit and errors of a cubic model for current 1.

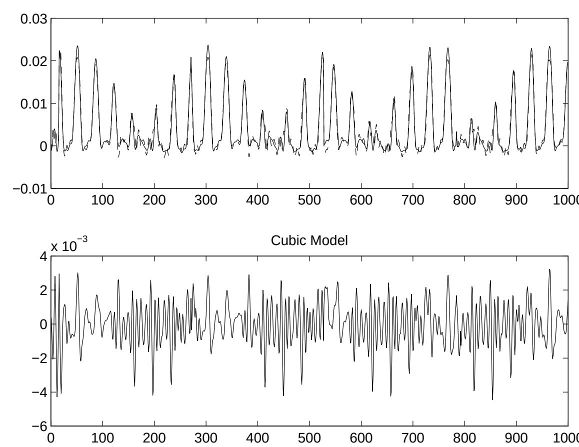

FIG. 6. The fit and errors of a cubic model for current 2.

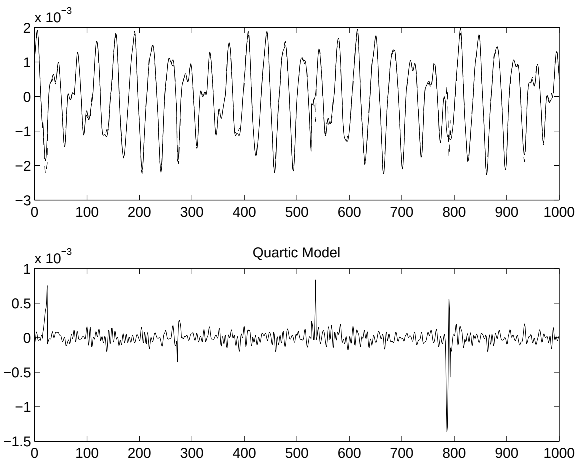

FIG. 7. The fit and errors of a quartic model for current 1.

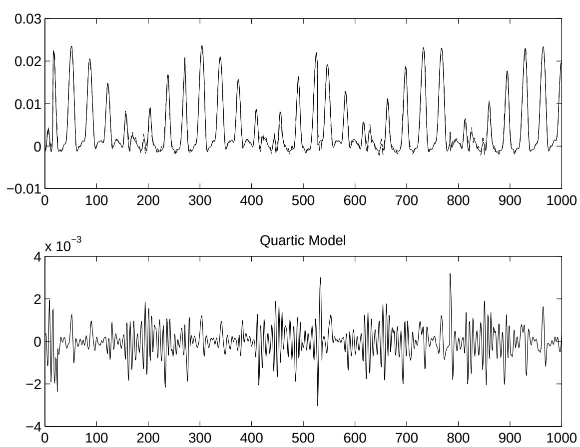

FIG. 8. The fit and errors of a quartic model for current 2.

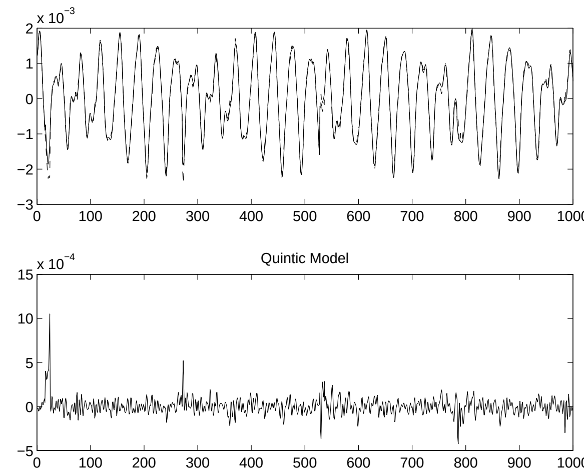

FIG. 9. The fit and errors of a quintic model for current 1.

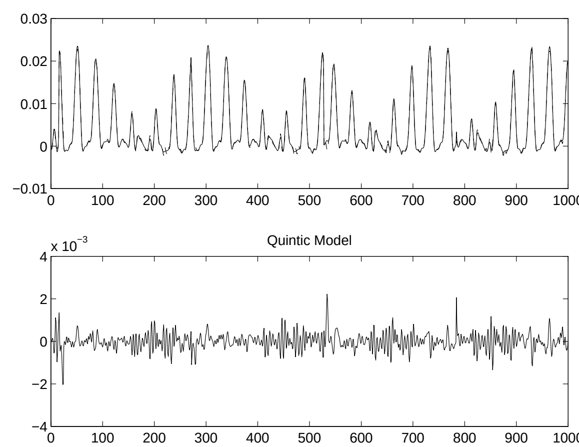

FIG. 10. The fit and errors of a quintic model for current 2.

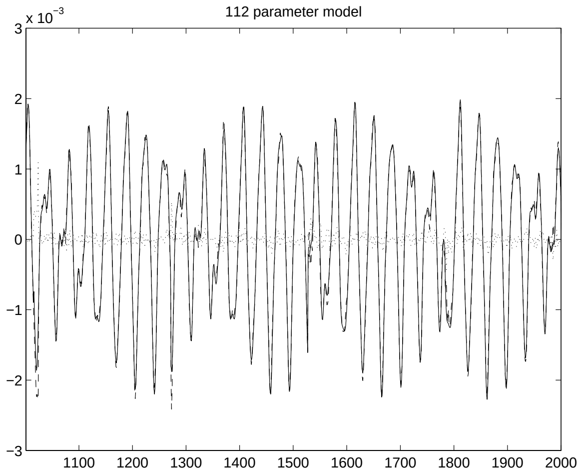

FIG. 11. The fit and errors of a 112 parameter model for current 1.

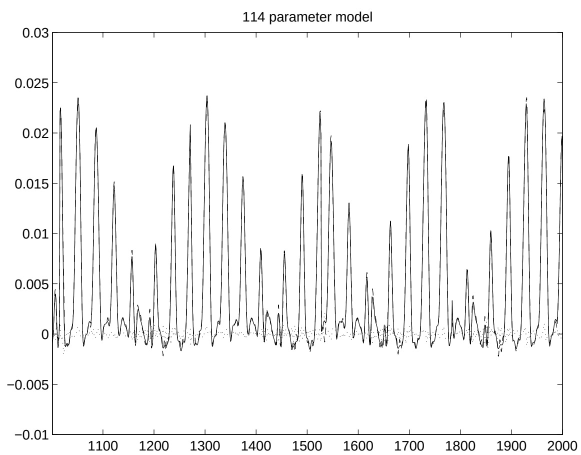

FIG. 12. The fit and errors of a 114 parameter model for current 2.

In Figures 1– 10 we show the results of reconstructing polynomial models of order one through to five. Table I gives the root–mean–sqare errors (divided by the standard deviation of the data to be predicted) of each of these fits. The modelling errors decrease as we increase the order of the polynomial as they should but it is interesting to note that there is a marked improvement when a cubic model is compared to the linear model. This suggests that the data is best modelled using nonlinear methods.

Model	Current 1	Current 2
Linear	0.4491	0.6882
Quadratic	0.2799	0.4144
Cubic	0.2373	0.1935
Quartic	0.1966	0.1056
Quintic	0.1421	0.0629

TABLE I. This table shows the error of the training fit using different orders of polynomials. The error is calculated to be the rms error divided by the standard deviation of the data to be predicted. We observe that the errors decrease as we increase the order of the polynomials. This is to be expected since we have increased the number of parameters to be fitted.

To investigate this further we shall use the subset selection and description length ideas of Judd and Mees [13] to perform the parameter estimation. The highest order we allow our polynomial models to have is five. The results are shown in Figures 11– 12 and the modelling errors are 0.1423 for current 1 using a 112 parameter model and 0.0630 for current 2 using a 114 parameter model. We notice that the description length method returned a model of many parameters indicating a nonlinear relationship between the voltages and currents. We also observe that both models used less parameters than the 126 available for a quintic polynomial model with comparable fitting errors.

B. Modelling using a few of the data sets

The results of the previous section suggest that it is appropriate to use nonlinear methods to describe the data. The work lacked an investigation of how general the reconstructed models were. That is, can the reconstructed model test well on other data sets. To study this we reconstruct a polynomial model as above but use only a few of the data sets to perform the training step. We then test the ability of the models to generalize to other data sets.

Recall at one bias point we have 15 data sets distinguished by the phase difference of the incidence voltages. We will label these data sets z1 through to z15. In the training stage we will use the data sets labelled z1, z4, z7, z10, z13 and z15. In the testing stage we will use the data sets labelled z3, z6, z9 and z12. As before we use the subset selection algorithm to reconstruct a polynomial model with maximum order of five. The lags and embedding dimensions also remain the same.

In Table II we show the results of training and testing a polynomial model on the above data sets. We see that the models test well on the unseen data.

	Current 1	Current 2
No. of Parms	112	108
Fit Error	0.1111	0.0452
z3 Test	0.0740	0.0452
z6 Test	0.0915	0.0439
z9 Test	0.0794	0.0467
z12 Test	0.0874	0.0429

TABLE II. The modelling and test errors of a polynomial model reconstructed using subset selection and description length. The training data used 6 data sets different from the 4 test data sets.

For comparison we have applied some radial basis modelling techniques to the same training and test data sets used above. Table III shows the preliminary results. We see similar results to the polynomial models but the errors are a bit higher. A more concerted effort could improve these results.

	Current 1	Current 2
No. of Parms	24	52
Fit Error	0.3122	0.1542
z3 Test	0.2650	0.1569
z6 Test	0.2659	0.1776
z9 Test	0.2345	0.1458
z12 Test	0.2289	0.1273

TABLE III. The modelling and test errors of a radial basis model reconstructed using subset selection and description length. The training data used 6 data sets different from the 4 test data sets. We see that the models generalize in the same way as the polynomial models albeit with a higher overall error.

IV. SUMMARY

In this preliminary report we have shown that measurements of a InP HEMT from a nonlinear network measurement system can be adequately modelled using nonlinear black–box behavioural models. We have demonstrated that the data we study is nonlinear and that only the present and past voltage values are needed to predict the present currents. The models were shown to test well on data sets other than those used to reconstruct the models. A brief comparison between polynomial models and radial basis models was carried out. We saw that both classes of models had similar generalisation properties but that overall the polynomial models had better test error performance. We limited this report to the study of data from one bias point.

V. ACKNOWLEDGEMENTS

D. Schreurs is supported by the Fund for Scientific Research-Flanders as a post-doctoral fellow. We thank M. Vanden Bossche for his help in understanding the theory and operation of the NNMS.

[1] W. J. Rugh, Linear System Theory (Prentice Hall, NJ, 1996).

[2] Neil A. Gershenfeld, The Nature of Mathematical Modeling (Cambridge University Press, 1999).

[3] M. Casdagli, A dynamical systems approach to modeling input-output systems, in Nonlinear Modeling and Forecasting, SFI Studies in the Sciences of Complexity, Proc. Vol XII, Eds. M. Casdagli and S. Eubank, Addison-Wesley, 1992.

[4] S. Haykin and J. Principe, Making sense of a complex world, IEEE Signal Processing Magazine, 66-81 (May 1998).

[5] J. Stark, Delay embeddings and forced systems, *J. Nonlinear Science*, **9** 255-332, 1999.

[6] G. F. Franklin, Feedback control of dynamic systems (Addison-Wesley, 1994)

[7] H. Kantz and T. Schreiber, Nonlinear time series analysis, (Cambridge University Press, 1997).

[8] David M. Walker and Nicholas B. Tufillaro, Phase space reconstruction using input-output time series data, *Phys. Rev. E* **60** (4), 4008-4013, 1999.

[9] David M. Walker, Reggie Brown, and Nicholas B. Tufillaro, Constructing transportable behavioral models for nonlinear electronic devices, *Phys. Letts A* (255) 4-6, 236-242, 1999.

[10] Jan Verspecht and Tom Van den Broeck, Calibrated vectorial nonlinear network analyzers, Microwave Symposium Digest of IEEE 1994 MTT-S International.

[11] C. Rhodes and M. Morari, Determining the model order of nonlinear input–output systems, *Proceedings of the American Control Conference*, Seattle, 2190–2195, 1995.

[12] L. Cao, A. Mees, K. Judd and G. Froyland, Determining the minimum embedding dimension of input–output time series data, *Int. J. Bif. Chaos*, 8(7):1491–1504, 1998.

[13] K. Judd and A. Mees, *Physica* **82D**, 426 (1995).